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Phenomenology of Non-Abelian Flat Directions in a Minimal Superstring Standard Model

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Abstract

Recently, we presented the first non-Abelian flat directions that produce from a heterotic string model solely the three-generation MSSM states as the massless spectrum in the observable sector of the low energy effective field theory. In this paper we continue to develop the systematic techniques for the analysis of nonrenormalizable superpotential terms and non-Abelian flat direction in realistic string models. Some of our non-Abelian directions were *F*-flat to *all* finite orders in the superpotential. We study for the same string model the varying phenomenologies resulting from a large set of such all-order flat directions. We focus on the quark, charged lepton, and Higgs doublet mass matrices resulting for our phenomenologically superior non-Abelian flat direction. We review and apply a string-related method for generating large mass hierarchies between MSSM generations, first discussed in string-derived flipped $SU(5)$ models, when all generational mass terms are of renormalizable or very low non-renormalizable order.

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1 Minimal Superstring Standard Models

Supersymmetry and heterotic string unification continue to be the only extensions of the Standard Model* that are motivated by a successful experimental prediction. Indeed, for over a decade now, the consistency of the MSSM gauge coupling unification with the low energy experimental data [1] has provided the most appealing hint for the validity of the big desert scenario and Planck scale unification [2]. On the other hand, heterotic string theory provides a consistent theoretical framework to study how the structure of the Standard Model may arise from Planck scale physics. The success of the MSSM gauge coupling unification implies that the string models that should be constructed are those that contain solely the MSSM spectrum in the effective low energy field theory below the string scale. The complete solution will then be achieved by embedding such models in M-theory and extrapolating to the strong coupling regime in which the string and MSSM unification scales coincide [3]. It is therefore remarkable that recently we were able to construct string models in which the only Standard Model charged fields, in the low energy effective field theory, coincide exactly with the matter content of the MSSM [4, 5, 6, 7]. The starting point for the construction of the first known example of a “Minimal Standard Heterotic-String Model,” (MSHSM) is the free fermionic string model first presented in ref. [8], referred to as the “FNY” model. The FNY observable gauge group is $SU(3)_C \times SU(2)_L \times U(1)_Y \times \prod_i U(1)_i$. As with all three-generations (2, 0) string-based $SU(3)_C \times SU(2)_L \times U(1)_Y$ models possessing free-fermions, free lattice bosons, or orbifolds, for internal degrees of freedom, FNY possesses an anomalous $U(1)_A$. Cancellation of this anomaly via the standard string mechanism and related appearance of the Fayet-Iliopoulos (FI) D -term results in the breaking of supersymmetry near the string scale, unless an appropriate set of fields carrying the anomalous charge take on vacuum expectation values (VEVs) that together cancel the effects of the FI term, while keeping the respective VEVs of all other non-anomalous D -terms at zero. F -flatness must also be retained up to an order in the superpotential that is consistent with observable sector supersymmetry being maintained down to near the electroweak (EW) scale. Depending on the string coupling strength, F -flatness cannot be broken by terms below eighteenth to twentieth order†.

The eventual determination of the set of fields chosen to acquire VEVs (a.k.a. the flat direction) in a given string model, will be fixed by non-perturbative effects. However, the set of perturbatively allowed flat directions can be determined. The varying phenomenology of these flat directions can be studied, or conversely, the subset of flat directions satisfying known phenomenological requirements can be focused on. Initially we investigated FNY flat directions composed solely of VEVs of non-Abelian singlet fields [4, 5, 6]. While these directions offered a good first step in constructing a physically viable MSHSM model, we found some short-comings to

*Standard Model here also refers to its extensions that include neutrino masses.

†As coupling strength increases, so does the required order of flatness.

this approach. Rather, the inclusion of hidden-sector non-Abelian fields was strongly suggested. The necessity to include non-Abelian flat directions was also suggested in analysis of other semi-realistic free fermionic string models [9]. These were, however, preliminary studies and the inclusion of the non-Abelian fields in the systematic analysis of the flat directions is still lacking. Thus, in [7] we began our study of FNY non-Abelian flat directions, which we continue in this paper. We emphasize that the main aim of the present paper is to continue to develop the techniques and methodology that are needed to confront string theory with the low energy experimental data. We therefore stress that we do not intend to suggest or imply that we will find the “theory of everything” through investigation of the FNY model or of its MSHSM-producing flat directions. Nevertheless, the case may still well be that some of the general properties and phenomenology of the true string vacuum will be gleaned in the process.

Our paper is organized as follows. First, in Section 2 we review the general structure of free fermionic heterotic string models. Then in Section 3, we summarize the generic properties of Abelian and non-Abelian flat directions and their application to heterotic string models. We also present and discuss some of the results of our recent non-Abelian flat direction search, including the constraints we placed on this search. Next, in Section 4 we delve into non-Abelian flat directions for the FNY model. In Section 5 we investigate the phenomenology associated with these directions. Specifically we investigate the three generation quark and lepton mass matrices, the Higgs effective μ -terms, and issue of proton decay in the FNY MSHSMs. One especially interesting feature we find is that, because of the several, initially massless, Higgs doublets in FNY, a large hierarchy between the three generations MSSM fields is possible even when the quark and lepton mass terms are all of very low order in the superpotential. While only one Higgs doublet pair eigenstate can survive in the low energy MSHSM effective field theory, that eigenstate may have several non-eigenstate Higgs doublets as unequally weighted components. Because of the unequal weighting, differing naturally by several orders of magnitude, and preferred coupling between generations and Higgs components, a large hierarchy can arise. In Section 6 we present our concluding remarks.

2 General structure of free fermionic models

In this section we briefly review the general features, which reveal why non-Abelian flat directions are in fact necessary for obtaining viable fermion mass spectrum in the realistic free fermionic models. The details of the spectrum and more elaborate discussions on the general structure of the models, and the general methodology of their analysis, are given in the references and are not repeated here.

The free fermionic models are constructed by specifying a set of boundary condition basis vectors for the world-sheet free fermions [10]. The basis vectors that generate the realistic models can be divided into two groups. The first consist of the

NAHE set[‡] [12], $\{\mathbf{1}, \mathbf{S}, \mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$, and correspond to $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold compactification, where the three sectors \mathbf{b}_1 , \mathbf{b}_2 and \mathbf{b}_3 correspond to the three twisted sectors of the orbifold models. The gauge group at this level is $SO(10) \times SO(6)^3 \times E_8$, and the models contain 48 multiplets in the **16** representation of $SO(10)$. The NAHE set divides the internal world-sheet fermions into several groups with the set of 12 internal world-sheet fermions, $\{y, w | \bar{y}, \bar{\omega}\}$, corresponding to the six dimensional ‘‘compactified space’’, the 16 complex fermionic states $\{\bar{\psi}^{1,\dots,5}, \bar{\eta}^{1,2,3}, \bar{\phi}^{1,\dots,8}\}$ corresponding to the gauge sector, and $\chi^{1,\dots,6}$ corresponding to the RNS fermions, of the orbifold model. The $\{y, w | \bar{y}, \bar{\omega}\}$ world-sheet fermions are further divided into three cyclically symmetric groups, which reflects the structure of the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold. The states from each of the twisted sectors, \mathbf{b}_1 , \mathbf{b}_2 or \mathbf{b}_3 are then charged with respect to one of these orbifold planes. The NAHE set is common to a large class of realistic free fermionic models. Among them are the flipped $SU(5)$ string models [11], the Pati-Salam string models [13], the String standard-like models [8, 14], and the left-right symmetric string models [15]. As we elaborate further below, this general $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold structure is the reason for the necessity to incorporate non-Abelian flat directions.

The second step in the basis construction consists of adding to the NAHE set three, or four, additional boundary condition basis vectors, typically denoted as $\{\alpha, \beta, \gamma\}$. The effect of these is to break the $SO(10)$ symmetry to one of its subgroups, and at the same time reduce the number of generations to three, one from each twisted sector \mathbf{b}_i ($i = 1, 2, 3$). There are several important features of this class of models. The first is that since the generations are obtained from the three distinct twisted sectors of the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold, each generation is charged with respect to a distinct set of horizontal charges, which also holds for the untwisted Higgs doublets. Thus, to generate fermion mass terms that mix between the generations we need fields that are charged simultaneously with respect to at least two orbifold planes. Furthermore, because the charges of the generations under the horizontal symmetries are $\pm 1/2$ also, the charges of the mixing fields should combine to $\pm 1/2$.

To understand how the fields with the required properties arise in the free fermionic models, one has to examine the hidden sector of the free fermionic string models. The hidden sector is obtained from the E_8 factor at the level of the NAHE set and is generated by the $\{\bar{\phi}^{1,\dots,8}\}$ right-moving world-sheet fermions. The addition of the vector 2γ to the NAHE set induces the symmetry breaking $E_8 \rightarrow SO(16)$. At the same time the sectors $\mathbf{b}_j + 2\gamma$ produces multiplets in the vectorial **16** representation of $SO(16)$. The basis vectors beyond the NAHE set further break the $SO(16)$ symmetry to one of its subgroups, and the hidden sector massless states from the sectors $\mathbf{b}_j + 2\gamma$ transform in non-Abelian representations of the unbroken subgroup. It is precisely these states that induce the mixing terms between the chiral states from the twisted sectors \mathbf{b}_i ($i = 1, 2, 3$). These mixing terms therefore have the generic form

[‡]This set was first utilized by Nanopoulos, Antoniadis, Hagelin and Ellis (NAHE) in the construction of the flipped $SU(5)$ [11]. NAHE=pretty in Hebrew.

$16_i 16_j 10 16_i 16_j \phi^n$, where the first two **16** are in the spinorial representation of the observable $SO(10)$, the **10** is in the vector representation of the observable $SO(10)$ and correspond to the untwisted Higgs fields, the last two **16** are in the vector representation of the hidden $SO(16)$, and ϕ^n is a combination of $SO(10) \times SO(16)$ scalar singlets. We therefore see that, in general, producing fermion mass matrices with nontrivial structures necessitates that the hidden sector non-Abelian states from the sectors $\mathbf{b}_j + 2\gamma$ obtain non-trivial VEVs.

3 Non-Abelian Flat Directions and Spacetime Supersymmetry

3.1 D - and F -Flatness Constraints

In [4, 5, 6, 7] we reviewed D - and F -flatness and their well known requirements for maintenance of spacetime supersymmetry. Thus, we will only summarize this here, but with a new emphasis on geometric interpretation of the $SU(2)$ non-Abelian VEVs.

Spacetime supersymmetry is broken in a model when the expectation value of the scalar potential,

$$V(\varphi) = \frac{1}{2} \sum_{\alpha} g_{\alpha} \left(\sum_{a=1}^{\dim(\mathcal{G}_{\alpha})} D_a^{\alpha} D_a^{\alpha} \right) + \sum_i |F_{\varphi_i}|^2 , \quad (3.1)$$

becomes non-zero. The D -term contributions in (3.1) have the form,

$$D_a^{\alpha} \equiv \sum_m \varphi_m^{\dagger} T_a^{\alpha} \varphi_m , \quad (3.2)$$

with T_a^{α} a matrix generator of the gauge group \mathcal{G}_{α} for the representation φ_m . The F -term contributions are,

$$F_{\Phi_m} \equiv \frac{\partial W}{\partial \Phi_m} . \quad (3.3)$$

The φ_m are (spacetime) scalar superpartners of the chiral spin- $\frac{1}{2}$ fermions ψ_m , which together form a superfield Φ_m . Since all of the D and F contributions to (3.1) are positive semidefinite, each must have a zero expectation value for supersymmetry to remain unbroken.

For an Abelian gauge group, the D -term (3.2) simplifies to

$$D^i \equiv \sum_m Q_m^{(i)} |\varphi_m|^2 \quad (3.4)$$

where $Q_m^{(i)}$ is the $U(1)_i$ charge of φ_m . When an Abelian symmetry is anomalous, that is, the trace of its charge over the massless fields is non-zero,

$$\text{Tr } Q^{(A)} \neq 0 , \quad (3.5)$$

the associated D -term acquires a Fayet-Iliopoulos (FI) term, $\epsilon \equiv \frac{g_s^2 M_P^2}{192\pi^2} \text{Tr } Q^{(A)}$,

$$D^{(A)} \equiv \sum_m Q_m^{(A)} |\varphi_m|^2 + \epsilon. \quad (3.6)$$

g_s is the string coupling and M_P is the reduced Planck mass, $M_P \equiv M_{Planck}/\sqrt{8\pi} \approx 2.4 \times 10^{18}$ GeV. It is always possible to place the total anomaly into a single $U(1)$.

The FI term breaks supersymmetry near the string scale,

$$V \sim g_s^2 \epsilon^2, \quad (3.7)$$

unless its can be cancelled by a set of scalar VEVs, $\{\langle \varphi_{m'} \rangle\}$, carrying anomalous charges $Q_{m'}^{(A)}$,

$$\langle D^{(A)} \rangle = \sum_{m'} Q_{m'}^{(A)} |\langle \varphi_{m'} \rangle|^2 + \epsilon = 0. \quad (3.8)$$

To maintain supersymmetry, a set of anomaly-cancelling VEVs must simultaneously be D -flat for all additional Abelian and the non-Abelian gauge groups,

$$\langle D^{i,\alpha} \rangle = 0. \quad (3.9)$$

The consistent solution of (3.4), when placed into (3.8), specifies the overall VEV “FI-scale”, $\langle \alpha \rangle$, of the model. A typical FNY value is $\langle \alpha \rangle \approx 7 \times 10^{16}$ GeV.

For the case of $SU(2)$, $T_a^{SU(2)}$ will take on the values of the three Pauli matrices,

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \quad (3.10)$$

Each component of the vector \vec{D} in this internal space will be the total, summed over all fields of the gauge group, “spin expectation value” in the given direction. Vanishing of the $\langle \vec{D} \cdot \vec{D} \rangle$ contribution to $\langle V \rangle$ demands that $SU(2)$ VEVs be chosen such that the total \hat{x} , \hat{y} , and \hat{z} expectation values are zero. The normalization length, $S^\dagger S$, of a “spinor” S will generally be restricted to integer units by Abelian D -flatness constraints from the Cartan sub-algebra and any extra $U(1)$ charges carried by the doublet (cf. Eq. 3.4 with $S^\dagger S$ playing the role of $|\varphi|^2$). Each spinor then has a length and direction associated with it and D -flatness requires the sum, placed tip-to-tail, to be zero. This reflects the generic non-Abelian D -flatness requirement that the norms of non-Abelian field VEVs are in a one-to-one association with a ratio of powers of a corresponding non-Abelian gauge invariant [16].

It will be useful to have an explicit (normalized to 1) representation for $S(\theta, \phi)$. This may be readily obtained by use of the rotation matrix,

$$R(\vec{\theta}) \equiv e^{-i \frac{\vec{\theta} \cdot \vec{\sigma}}{2}} = \cos\left(\frac{\theta}{2}\right) - i \hat{\theta} \cdot \vec{\sigma} \sin\left(\frac{\theta}{2}\right), \quad (3.11)$$

to turn $\begin{pmatrix} 1 \\ 0 \end{pmatrix} \equiv |+\hat{z}\rangle$ through an angle θ about the axis $\hat{\theta} = -\hat{i}\sin(\phi) + \hat{j}\cos\phi$. The result, $\begin{pmatrix} \cos\frac{\theta}{2} \\ \sin\frac{\theta}{2}e^{i\phi} \end{pmatrix}$, is only determined up to a phase and the choice

$$S(\theta, \phi) \equiv \begin{pmatrix} \cos\frac{\theta}{2}e^{-i\frac{\phi}{2}} \\ \sin\frac{\theta}{2}e^{+i\frac{\phi}{2}} \end{pmatrix} \quad (3.12)$$

will prove more convenient in what follows. Within the range of physical angles, $\theta = 0 \rightarrow \pi$ and $\phi = 0 \rightarrow 2\pi$, each spinor configuration is unique (excepting ϕ phase freedom for $\theta = 0, \pi$) and carries a one-to-one geometrical correspondence. Up to a complex coefficient, the most general possible doublet is represented.

A non-trivial superpotential W additionally imposes numerous constraints on allowed sets of anomaly-cancelling VEVs, through the F -terms in (3.1). F -flatness (and thereby supersymmetry) can be broken through an n^{th} -order W term containing Φ_m when all of the additional fields in the term acquire VEVs,

$$\langle F_{\Phi_m} \rangle \sim \langle \frac{\partial W}{\partial \Phi_m} \rangle \sim \lambda_n \langle \varphi \rangle^2 \left(\frac{\langle \varphi \rangle}{M_{\text{str}}} \right)^{n-3}, \quad (3.13)$$

where φ denotes a generic scalar VEV. If Φ_m also carries a VEV, then supersymmetry can be broken simply by $\langle W \rangle \neq 0$ [§].

3.2 World Sheet Selection Rules

To survive in a generic field theory, a term contributing to the potential need only satisfy the sum of all charges equal to zero (thus ensuring conservation of each Abelian charge) and appropriate gauge invariant pairings of non-Abelian fields. String theory, on the other hand, is much more restrictive, demanding that each collection of fields pass a “picture-changing” test in order to conserve global worldsheet and Ising charges and retain modular invariance. This extra constraint is beneficial to our program by removing, at an early stage, many W - and F -terms which would otherwise be dangerous to supersymmetry.

In free fermionic heterotic string theory, extra world-sheet fermions are used to cancel the four-dimensional conformal anomaly. Fermionic boundary conditions define a given model and physical states may be produced out of either the Ramond (twisted) or Neveu-Schwarz (untwisted) sectors. In $N = 1$ supersymmetric models, six of the real left-moving fermions are bosonized, creating three $U(1)$ categories into which all physical states are separated. General selection rules predicting nonvanishing correlators have been formulated in [17] and in [18]. For completeness and because they are referenced in Section 3.4, these rules are here (with alterations in form) restated. The letter n will designate the total number of fields in a term.

[§]The lower the order of an F -breaking term, the closer the supersymmetry breaking scale is to the string scale.

1. Ramond fields must be distributed equally, mod 2, among all categories, *and*
2. For $n = 3$, either :
 - (a) There is 1 field from each R category.
 - (b) There is 1 field from each NS category.
 - (c) There are $2R$ and $1NS$ in a single category.
3. For $n > 3$:
 - (a) There must be at least $4R$ fields.
 - (b) All R fields may not exist in a single category.
 - (c) If $R = 4$, then only permutations of $(2_R, 2_R, n - 4_{NS})$ are allowed.
 - (d) If $R > 4$, then no NS are allowed in the maximal R category (if one exists).

3.3 Non-Abelian Flat Directions and Self-Cancellation

In [5] we classified MSSM producing singlet field flat directions of the FNY model and in [6] we studied the phenomenological features of these singlet directions. Our past investigations suggested that for several phenomenological reasons, including production of viable three generation quark and lepton mass matrices and Higgs $h\bar{h}$ mixing, non-Abelian fields must also acquire FI-scale VEVs.

In our prior investigations we generally demanded “stringent” flatness. That is, we forced each superpotential term to satisfy F -flatness by assigning no VEV to at least two of the constituent fields. While the absence of any non-zero terms from within $\langle F_{\Phi_m} \rangle$ and $\langle W \rangle$ is clearly sufficient to guarantee F -flatness along a given D -flat direction, such stringent demands are not necessary. Total absence of these terms can be relaxed, so long as they appear in collections which cancel among themselves in each $\langle F_{\Phi_m} \rangle$ and in $\langle W \rangle$. It is desirable to examine the mechanisms of such cancellations as they can allow additional flexibility for the tailoring of phenomenologically viable particle properties while leaving SUSY inviolate. It should be noted that success along these lines may be short-lived, with flatness retained in a given order only to be lost at one slightly higher.

Since Abelian D -flatness constraints limit only VEV magnitudes, we are left with the gauge freedom of each group (phase freedom, in particular, is ubiquitous) with which to attempt a cancellation between terms (whilst retaining consistency with non-Abelian D -flatness). However, it can often be the case that only a single term from W becomes an offender in a given $\langle F_{\Phi_m} \rangle$ (cf. Table 1B). If a contraction of non-Abelian fields (bearing multiple field components) is present it may be possible to effect a *self-cancellation* that is still, in some sense, “stringently” flat.

Near the string scale the complete FNY gauge group is

$$[SU(3)_C \times SU(2)_L \times U(1)_C \times U(1)_L \times U(1)_A \times \prod_{i=1'}^{5'} U(1)_i \times U(1)_4]_{\text{obs}} \times \\ [SU(3)_H \times SU(2)_H \times SU(2)_{H'} \times U(1)_H \times U(1)_7 \times U(1)_9]_{\text{hid}}. \quad (3.14)$$

The FNY non-Abelian hidden sector fields are triplets of $SU(3)_H$ or doublets of $SU(2)_H$ or $SU(2)_{H'}$. Self-cancellation of F -terms, that would otherwise break observable sector supersymmetry far above the electro-weak scale, might be possible for flat directions containing such doublet or triplet VEVs. Since intermediate scale $SU(3)_H$ triplet/anti-triplet condensates are more likely to produce viable observable sector electro-weak scale supersymmetry breaking than are their $SU(2)_{H'}$ counterparts, we focus herein on non-Abelian directions containing doublet, but not triplet, FI-scale VEVs.

Whenever “spinors” of $SU(2)$ appear in W , they are not of the form $S^\dagger S$, but rather are in the antisymmetric contraction

$$S_1 \cdot S_2 \equiv S_1^T \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} S_2. \quad (3.15)$$

This form, which avoids complex conjugation and thus satisfies the requirement of analyticity, is also rotationally (gauge) invariant as can be verified using $\{\sigma_i, \sigma_j\} = 2\delta_{ij}$, $[\sigma_2, \sigma_2] = 0$, and Eqs. (3.10, 3.11) :

$$\sigma_2 R(\vec{\theta}) = R^*(\vec{\theta}) \sigma_2 \quad (3.16)$$

$$S'_1 \cdot S'_2 = (RS_1)^T (i\sigma_2) (RS_2) = S_1^T (i\sigma_2) (R^\dagger R) S_2 = S_1 \cdot S_2 \quad (3.17)$$

From Eq. (3.12), the general form of such a contraction may be written explicitly as

$$S(\theta, \phi) \cdot S(\Theta, \Phi) = -\sin\left(\frac{\theta - \Theta}{2}\right) \cos\left(\frac{\phi - \Phi}{2}\right) - i \sin\left(\frac{\theta + \Theta}{2}\right) \sin\left(\frac{\phi - \Phi}{2}\right). \quad (3.18)$$

The magnitude of this term must be a purely geometrical quantity and can be calculated as

$$|S(\hat{n}) \cdot S(\hat{N})| = \sqrt{\frac{1 - \hat{n} \cdot \hat{N}}{2}} = \sin\left(\frac{\delta}{2}\right), \quad (3.19)$$

where $\delta(0 \rightarrow \pi)$ is the angle between \hat{n} and \hat{N} . The absence of a similar concise form for the phase is not a failing of rotational invariance, but merely an artifact of the freedom we had in choosing (3.12). Self-cancellation of this term is independent of the spinor’s lengths and demands only that their spatial orientations be parallel[¶]. The same conclusion is reached by noting that antisymmetrizing the equivalent (or

[¶]The contraction of a field with itself vanishes trivially.

proportional) spinors yields a null value. VEVs satisfying this condition are clearly not D -consistent unless other non-Abelian VEVed fields also exist such that the *total* “spin” vector sum remains zero^{||}. To examine generic cases of cancellation between multiple terms, the full form of (3.18) is needed.

As an important special case, consider the example of a superpotential term $\phi_1 \dots \phi_n S_1 S_2 S_3 S_4$ ^{**} with ϕ_n Abelian. This is shorthand for an expansion in the various pairings of non-Abelian fields,

$$W \propto \phi_1 \dots \phi_n \{ (S_1 \cdot S_2)(S_3 \cdot S_4) + (S_2 \cdot S_3)(S_1 \cdot S_4) + (S_3 \cdot S_1)(S_2 \cdot S_4) \}, \quad (3.20)$$

Broadly, we notice that:

- Whenever each term holds the same field set the spinors may be treated as normalized to one, bringing any larger magnitudes out front as overall factors. Furthermore, since S^T appears but never S^\dagger , the same can be done with any phase selections.
- Since the contractions are antisymmetric, sensible interpretation of terms with multiple factors demands the specification of an ordering.

The appropriate ordering, or equivalently the choice of relative signs, for (3.20) is such to ensure *total* antisymmetrization. When (3.20) is explicitly evaluated using the previously established formalism it is seen to vanish identically for *all* field values. The calculation is simplified without loss of generality by taking $\theta_1 = \phi_1 = \phi_2 = 0$. We emphasize the distinction between this identical exclusion from the superpotential and cancellations which exist only at the vacuum expectation level. W -terms with 6 non-Abelian fields are formed with factors of (3.20) and also vanish, as do all higher order terms.

Even safe sectors of W (in particular with $\langle \Phi_m \rangle = 0$) may yield dangerous $\langle F_{\Phi_m} \rangle \equiv \langle \frac{\partial W}{\partial \Phi_m} \rangle$ contributions. The individual F -terms may be separated into two classes based on whether or not Φ_m is Abelian. For the case of Φ_m non-Abelian, $\langle F_{\Phi_m} \rangle$ is itself a doublet. As a note, terms like $\langle F_{S_4} \rangle \equiv \langle \frac{\partial W}{\partial S_4} \rangle$ which *would have* arisen out of (3.20) are cyclically ordered and also vanish identically.

3.4 Cancellation in Terms Bearing Φ_4 VEVs

The four fields designated as ‘ (Φ_4) ’ form a special system which merits some focused discussion. The pair (Φ_4, Φ'_4) carry identical charges for all $U(1)$ ’s and we must then consider a separate term for each way to divide the field content. However, Φ_4 is from Neveu-Schwarz category two while Φ'_4 is from category three and picture-changing may disallow one or both fields from a term. The same holds for their

^{||}In the notation of [6], taking a single sign for each of the $s_{k'}$ is a special case of non-Abelian self-cancellation, as is $\sum_{k=1}^p n_k s_k = 0$ a special case of the D -constraint.

^{**}Here and in the following discussion we consider the doublets of a single symmetry group.

oppositely-charged vector partners. Furthermore, it is phenomenologically indicated that providing FI-scale VEVs to *all four* fields will decouple fractionally charged exotics to this heavy mass scale. This results, selection rules permitting, in infinite towers of ‘new’ potentially dangerous charge conserving terms formed from each piece of W by adding N of (Φ_4, Φ'_4) and N of $(\bar{\Phi}_4, \bar{\Phi}'_4)$. D -flatness (3.4) cannot uniquely fix the VEV scales, but demands only that

$$|\langle \Phi_4 \rangle|^2 + |\langle \Phi'_4 \rangle|^2 - (|\langle \bar{\Phi}_4 \rangle|^2 + |\langle \bar{\Phi}'_4 \rangle|^2) = a \times |\langle \alpha \rangle|^2, \quad (3.21)$$

where a is an integer (possibly negative) and $\langle \alpha \rangle$ is the FI-scale for a given flat direction. Similar effects occur whenever the vector partner of any field takes on a VEV. An additional constraint [4] specific to the (Φ_4) system is

$$(\langle \Phi_4 \rangle \langle \bar{\Phi}'_4 \rangle + \langle \bar{\Phi}_4 \rangle \langle \Phi'_4 \rangle) = 0. \quad (3.22)$$

F -flatness is destroyed by a tadpole term unless this is obeyed.

Using the rules stated in Section 3.2, we will now systematically discuss when picture-changing allows a (Φ_4) tower to be added onto valid existing terms in W .

(A) If $n = 3$,

then there is no tower because no addition of purely Neveu-Schwarz fields can lead to an acceptable term (with at least four Ramond fields).

(B) If $R = 4$, $(n - 4_{NS}, 2_R, 2_R)$, with R fields in categories 2 and 3,

then there is no tower because NS fields may not be added to categories 2 or 3.

(C) If $R = 4$ with R fields in category 1 *or* $R > 4$ with an R maximum in 2 or 3,

then there is a tower but only $(\Phi_4, \bar{\Phi}_4)$ or $(\Phi'_4, \bar{\Phi}'_4)$ can participate in it and just a single term exists at each order.

(D) If $R > 4$ and there is no maximal R category *or* category 1 is maximal,
then the full tower is present without restrictions.

If vector partners of other fields have VEVs then *each* term in their tower is to be seen as a potential base point for its *own* (Φ_4) tower.

The presence of all these similar terms begs an investigation of when a self-cancellation is possible. As caveats, we will

- only consider cancellation within a single “base term” of W , and only within a single order at a time,
- naively assume a uniform non-renormalizable coupling for each term in a given cancellation.

Of the scenarios listed above, only (D) has multiple terms from (Φ_4) 's at a given order and can sustain self-cancellation. However, if the base order W term has no explicit (Φ_4) factors then it cannot participate and the effort is useless. We will now undertake a detailed look at the more interesting case of m fields from (Φ_4, Φ'_4) in the base term of a tower. The discussion for $(\bar{\Phi}_4, \bar{\Phi}'_4)$ follows directly. In general, to effect a cancellation at this lowest level, it is required that

$$\sum_{i=0}^m \langle \Phi_4 \rangle^i \langle \Phi'_4 \rangle^{m-i} = 0. \quad (3.23)$$

At an order $(2N)$ above the base order,

$$\sum_{i=0}^{m+N} \sum_{j=0}^N [\langle \Phi_4 \rangle^i \langle \Phi'_4 \rangle^{(m+N)-i}] [\langle \bar{\Phi}_4 \rangle^j \langle \bar{\Phi}'_4 \rangle^{N-j}] = 0 \quad (3.24)$$

$$= \sum_{i=0}^{m+N} \sum_{j=0}^N [\langle \Phi_4 \rangle^i \langle \Phi'_4 \rangle^{m-i}] \langle \Phi'_4 \rangle^N [\langle \bar{\Phi}_4 \rangle^j \langle \bar{\Phi}'_4 \rangle^{N-j}]. \quad (3.25)$$

Factoring (3.23) out of (3.25) and shifting the i index, leaves $\sum_{i=1}^N \sum_{j=0}^N [\langle \Phi_4 \rangle^{i+m} \langle \Phi'_4 \rangle^{N-i}] [\langle \bar{\Phi}_4 \rangle^j \langle \bar{\Phi}'_4 \rangle^{N-j}]$, or for $N = 1$,

$$\langle \Phi_4 \rangle^{m+1} (\langle \bar{\Phi}_4 \rangle + \langle \bar{\Phi}'_4 \rangle) = 0. \quad (3.26)$$

To satisfy Eq. (3.23), we must take *both* or *neither* of $(\langle \Phi_4 \rangle, \langle \Phi'_4 \rangle)$ as zero. Taking neither leads to the restriction from (3.26) that $\langle \bar{\Phi}_4 \rangle = -\langle \bar{\Phi}'_4 \rangle$ and, unless we choose $\langle \bar{\Phi}_4 \rangle = 0$, this leads via (3.22) to $\langle \Phi_4 \rangle = \langle \Phi'_4 \rangle$, which is clearly inconsistent with (3.23). If we do take $\langle \bar{\Phi}_4 \rangle = 0$, then the expectation value of the tower does vanish and the solution is consistent as long as 'a' from Eq. (3.21) is positive. If a equals zero, the only consistent solution is the trivial $\langle \Phi_4 \rangle = \langle \Phi'_4 \rangle = \langle \bar{\Phi}_4 \rangle = \langle \bar{\Phi}'_4 \rangle = 0$. If a is negative, then $(\langle \Phi_4 \rangle, \langle \Phi'_4 \rangle)$ must be zero and since all terms in the tower contain these fields, its expectation value vanishes as well. This discussion has been for towers added to F -dangerous terms only. For the case of a non-vanishing contribution to $\langle W \rangle$ itself, derivatives with respect to the (Φ_4) 's must be considered and $(\langle \Phi_4 \rangle, \langle \Phi'_4 \rangle)$ are now *demanded* to be zero (which is inconsistent for a positive).

It seems that, at least within the context of our assumptions, only fairly simple solutions exist which are at odds with the phenomenological imperative to assign FI-scale VEVs to all four of the (Φ_4) . It also seems likely that removing those assumptions would only serve to make cancellation more difficult.

Finally, we will briefly take up the question of when and how the lowest level term, (3.23) itself, can be zero. A natural choice seems to be $|\langle \Phi_4 \rangle|^2 = |\langle \Phi'_4 \rangle|^2$, although in this scenario the original motivation for FI-scale VEVs has been largely forsaken. It might be that the resulting VEV²s from (3.21) are fractional parts of $|\langle \alpha \rangle|^2$, but

this would be no more strange than the integer portions currently assigned to other fields. Putting $\langle \Phi_4 \rangle = e^{i\theta} \langle V \rangle$ and $\langle \Phi'_4 \rangle = \langle V \rangle$ into Eq. (3.23) yields

$$\langle V \rangle^m \sum_{\gamma=0}^m e^{i\gamma\theta} = 0. \quad (3.27)$$

‘Tip-to-tail’ solutions in the complex plane are always possible, for example, with $\theta = \frac{2\pi}{m+1}$. However, it must be kept in mind that a single set of VEVs for the (Φ_4) must be chosen *simultaneously* for the entire ‘flat direction’ and there may be issues of compatibility between the various incarnations of Eq. (3.23).

4 Minimal Standard Heterotic-String Model Non-Abelian Flat Directions

Our initial systematic search for MSSM-producing stringent flat directions revealed four singlet directions that were flat to all order, one singlet direction flat to twelfth order, and numerous singlet directions flat only to seventh order or lower [5]. For these directions renormalizable mass terms appeared for one complete set of up-, down-, and electron-like fields and their conjugates. However, the apparent top and bottom quarks did not appear in the same $SU(2)_L$ doublet. Effectively, these flat directions gave the strange quark a heavier mass than the bottom quark. This inverted mass effect was a result of the field Φ_{12} receiving a VEV in all of the above direction.

We thus performed a search for MSSM-producing singlet flat directions that did not contain $\langle \Phi_{12} \rangle$. None were found. This, in and of itself, suggests the need for non-Abelian VEVs in more phenomenologically appealing flat directions. Too few first and second generation down and electron mass terms implied similarly.

One interesting aspect of the singlet flat direction mass matrices was the implication that the vector partner of Φ_{12} should take on a VEV, but both renormalizable flatness constraints and viable Higgs μ -terms required $\langle \bar{\Phi}_{12} \rangle$ to be many orders of magnitude below the FI VEV scale of the flat direction fields. This possibility will be reconsidered in our three generation mass matrix discussions in Section 5.

The results of our non-Abelian flat direction search, which we summarize in Tables 1A and 1B, proved interesting. ‘FDNA#’ designations from [7] are continued. In general, the number of distinct $U(1)$ charges in a model is equal to the minimum number of fields, charged *independently* under those $U(1)$ ’s, which must take VEVs in order for dangerous gauge invariant terms to emerge in $\langle W \rangle$. In fact, FDNA5, which has only nine fields taking VEVs (counting (Φ_4) once), achieves its F -flatness in just this way, unable to obey gauge constraints without two or more unVEVed fields*. However, D -flat directions of this sort will be very limited. Equation (3.4),

*The third order (Φ_4) term mentioned near Eq. (3.22) will appear, but it is cancelled.

which does not restrict the anomalous charge and thus imposes one fewer condition than W , allows maximally[†] a single solution at this level per set of VEVed fields. Alternatively, if you are “one field short”, the inclusion of a field with no VEV may[†] produce one gauge invariant F -term. The remaining directions listed in Table 1A, besides FDNA(5+8), denoting the specific combination 18 FDNA5 + 1 FDNA8, are of this sort. They contain ten fields with VEVs and are dangerous to SUSY only through isolated $\langle F \rangle$ terms, but never $\langle W \rangle$. Since there are a finite number of fields which produce $\langle F_{\varphi_i} \rangle$ contributions, if all possibilities have been exhausted and the direction remains safe, the result holds to all orders.

We discovered several MSSM-producing D -flat directions that are stringently F -flat to at least seventh order and that satisfy the top-bottom $SU(2)$ doublet requirement of $\langle \Phi_{12} \rangle = 0$. Seven of these directions are F -flat to all finite order. The simplest, FDNA5, is the only direction for which no F -breaking terms appear in a gauge invariant superpotential. For both FDNA8 and FDNA9, there is just one respective dangerous (eleventh order) gauge invariant term and it is not invariant under the picture-changed global worldsheet symmetries of the string theory[‡]. Thus, neither term will appear in the low energy effective field theory superpotential. For FDNA6, one twelfth, two thirteenth, one seventeenth, and one eighteenth order dangerous gauge invariant terms do exist. Again, stringy worldsheet charge requirements eliminate terms, but this time one of the thirteenth order offenders remains. However, it is of the form of Eq. (3.20), containing four non-Abelian fields, and vanishes identically. Likewise, FDNA7, FDNA10, and FDNA17 are flat to all order thanks to the selection rules and excessive non-Abelian field content.

The remaining VEV directions listed in Table 1A are broken at finite order. FDNA11 and FDNA12 contain dangerous ninth order terms. The *same* eighth order term spoils each of FDNA13, FDNA14, FDNA15, and FDNA16. FDNA18 F -flatness breaks at seventh order. Interestingly, in all of these cases there is only ever a single term still standing, but for none of them do D -flatness requirements permit a consistent non-Abelian self-cancellation solution.

We also include FDNA(5+8) in the tables, which breaks F -flatness at only fifth order, but has none the less provided an interesting study. In particular, we have found terms ranging from fourth to tenth order which *do* experience a self-cancellation. It will be discussed shortly in much greater detail.

5 Flat Direction Phenomenology

[†]Although an *algebraic* gauge invariant solution can always be generated, the restriction to integer field number coefficients and signs which respect unassigned vector partner VEVs, can greatly reduce the incidence of *physical* solutions from the start.

[‡] This can be shown [19] based on the rules for allowed picture-changed global worldsheet charge combinations presented in [17, 18] and summarized in section 3.2.

5.1 Higgs Mass Matrix and μ -Terms

The FNY model contains four fields, $h_{1,2,3}$ and $h_4 \equiv H_{41}$, that can play the role of the MSSM Higgs doublet h and four corresponding fields, $\bar{h}_{1,2,3}$ and $\bar{h}_4 \equiv H_{34}$, that can play the role of the Higgs doublet \bar{h} . Higgs mass “ μ -terms” take the form of one field from each of these sets plus one or more fields which take a VEV. Collectively, they may be expressed in matrix form as the scalar contraction

$$h_i M_{ij} \bar{h}_j. \quad (5.1)$$

Specifically, we are interested in developing unique massless eigenstates h and \bar{h} formed from linear combinations of the h_i and \bar{h}_i respectively,[§]

$$h = \frac{1}{n_h} \sum_{i=1}^4 c_i h_i; \quad \bar{h} = \frac{1}{n_{\bar{h}}} \sum_{i=1}^4 \bar{c}_i \bar{h}_i, \quad (5.2)$$

with normalization factors $n_h = \sqrt{\sum_i (c_i)^2}$, and $n_{\bar{h}} = \sqrt{\sum_i (\bar{c}_i)^2}$. These combinations will then in turn establish the quark and lepton mass matrices.

However, the physical mass terms are those appearing in the scalar Lagrangian. They do not exist in the superpotential itself, but rather arise out of it in the form $|\frac{\partial W}{\partial \Phi_m}|^2$. Thus, the mass terms relevant to the fields (\bar{h}_j) are produced from derivatives of Eq. (5.1) with respect to the h_i , and may be written as

$$\sum_{i=1}^4 |M_{ij} \bar{h}_j|^2 = (\bar{h})^\dagger M^\dagger M (\bar{h}). \quad (5.3)$$

The matrix $M^\dagger M$ will then yield the m_h^2 eigenvalues when diagonalized. Since the matrix is Hermitian these will be real, although they do remain sensitive to phase selections within M . Furthermore, since the eigenvectors are orthogonal (or can be chosen so with degeneracy), they may be used, once normalized, to construct a unitary matrix $U \equiv \begin{pmatrix} u_1 & u_2 & \cdots \\ \downarrow & \downarrow & \end{pmatrix}$. The diagonalization of $M^\dagger M$ proceeds by inserting factors[¶] of $\mathbf{1} = (UU^\dagger)$ into (5.3), $(\bar{h})^\dagger U U^\dagger M^\dagger M U U^\dagger (\bar{h})$, so as to facilitate a grouping into four distinct mass values without disrupting the actual collection of terms. The right-most matrix, $U^\dagger \equiv \begin{pmatrix} u_1^* & \rightarrow \\ u_2^* & \rightarrow \\ \cdots & \end{pmatrix}$, serves to receive column vectors and project out their diagonal coefficients, while the adjacent factor of U ensures that

[§]The possibility of linear combinations of MSSM doublets forming the physical Higgs is a feature generic to realistic free fermionic models.

[¶] The two instances of (UU^\dagger) must be identically composed if the defining relation, $(U^\dagger M U)(U^\dagger |\Lambda\rangle) = \lambda(U^\dagger |\Lambda\rangle)$, is to hold for the diagonal basis.

vectors from the diagonal basis properly interpolate to the original^{||} before passing through $M^\dagger M$. Similarly, the (h_i) mass terms are

$$\sum_{j=1}^4 |h_i M_{ij}|^2 = (h)^T M M^\dagger (h)^*. \quad (5.4)$$

The MM^\dagger eigenvectors, denoted as (v_i) , compose a matrix we will call U' . Finally, the massless physical Higgs doublets may be expressed as

$$h = \sum_{i=1}^4 v_i^{(0)} h_i; \quad \bar{h} = \sum_{i=1}^4 u_i^{*(0)} \bar{h}_i, \quad (5.5)$$

the zero-mass elements of $(U'^T h)$ and $(U^\dagger \bar{h})$ respectively.

We have found that both singlet and non-Abelian MSSM-producing flat directions necessarily contain Φ_{23} , H_{31}^s , and H_{38}^s VEVs. Together these three VEVs produce four terms in the Higgs mass matrix: $h_3 \bar{h}_2 \langle \Phi_{23} \rangle$, $h_2 \bar{h}_4 \langle H_{31} \rangle$, $h_4 \bar{h}_3 \langle H_{38} \rangle$, and $h_4 \bar{h}_4 \langle H_{31} \rangle$. When these are the only non-zero terms in the matrix, the massless Higgs eigenstates simply correspond to $c_1 = \bar{c}_1 = 1$ and $c_j = \bar{c}_j = 0$ for $j = 2, 3, 4$. In this case all possible quark and lepton mass terms of the form $Q_m u_n^c \bar{h}_j$, $Q_m d_n^c h_j$, $Q_m e_n^c h_j$, $L_m N_n^c h_j$, $L_m L_n h_i h_j$, where $m, n \in \{1, 2, 3\}$, $j \in \{2, 3, 4\}$, and $i \in \{1, 2, 3, 4\}$ decouple from the low energy MSSM effective field theory. However, when one or more of the c_j or \bar{c}_j are non-zero, then some of these terms are not excluded and provide addition quark and lepton mass terms. In such terms, the Higgs components can be replaced by their corresponding Higgs eigenstates along with a weight factor,

$$h_i \rightarrow \frac{c_i}{n_h} h; \quad \bar{h}_i \rightarrow \frac{\bar{c}_i}{n_{\bar{h}}} \bar{h}. \quad (5.6)$$

Thus, in string models such as the FNY, two effects can contribute to inter-generational (and intra-generational) mass hierarchies: generic suppression factors of $\frac{\langle \phi \rangle}{M_P}$ in non-renormalizable effective mass terms and $\frac{c_i}{n_h}$ or $\frac{\bar{c}_i}{n_{\bar{h}}}$ suppression factors. This means a hierarchy of values among the c_i and/or among the \bar{c}_i holds the possibility of producing viable inter-generational $m_t : m_c : m_u = 1 : 7 \times 10^{-3} : 3 \times 10^{-5}$ mass ratios even when all of the quark and lepton mass terms are of renormalizable or very low non-renormalizable order order. Note that more than one generation of such low order terms *necessitates* a hierarchy among the c_i and \bar{c}_i . Generational hierarchy via suppression factor in Higgs components was first used in free fermionic models of the flipped $SU(5)$ class, see for example [20].

In Appendix B, we have provided all possible quark and electron-like mass terms through eighth order, for which the fourth through last fields in these terms must acquire a VEV. The FNY up- and down-like renormalizable terms are

$$\bar{h}_1 Q_1 u_1^c, \quad h_2 Q_2 d_2^c, \quad h_3 Q_3 d_3^c. \quad (5.7)$$

^{||} This is a statement of the completeness relation, $\sum |\Lambda\rangle\langle\Lambda| = \mathbf{1}$

Since \bar{h}_1 is either the only component or a primary component in \bar{h} , the top quark is necessarily contained in Q_1 and u_1^c is the primary component of the left-handed anti-top mass eigenstate. Thus, as we have already discussed, the bottom quark must be the second component of Q_1 . Since there are no renormalizable $h_i Q_1 d_m^c$ terms in Eq. (5.7), a bottom quark mass that is hierarchically larger than the strange and down quark masses requires that

$$\frac{|c_{j=2,3}|}{n_h} \ll 1. \quad (5.8)$$

Non-zero $c_{j=2,3}$ satisfying Eq. (5.8) could, perhaps, yield viable strange or down mass terms.

The first possible bottom mass term appears at fourth order,

$$h_4 Q_1 d_3^c H_{21}^s. \quad (5.9)$$

Realization of the bottom mass via this term would require h to contain a component of h_4 and for H_{21}^s to acquire a VEV. Of our flat directions, only FDNA8 and FDNA(5+8) give H_{21}^s a VEV. Of these two, only FDNA(5+8) embeds part of h_4 in h .

The physical mass ratio of the top and bottom quark is of order $\sim 3 \times 10^{-2}$. In free fermionic models there is apparently no significant suppression to the numeric value of an effective third order superpotential coupling constant, $\lambda_3^{\text{eff}} = \lambda_4 \langle \phi \rangle$, originating from a fourth order term. Hence, a reasonable top to bottom mass ratio would imply

$$\frac{|c_2|}{n_h}, |c_3| n_h \ll \frac{|c_4|}{n_h} \sim 10^{-2 \text{ to } -3} \quad (5.10)$$

when $\frac{|\bar{c}_1|}{n_{\bar{h}}} \sim 1$ and $\langle h \rangle \sim \langle \bar{h} \rangle$.

The next possible higher order bottom mass terms do not occur until sixth order:

$$\begin{aligned} h_2 Q_1 d_3^c \Phi_{13} V_2^s V_{21}^s &+ h_2 Q_1 d_3^c \Phi_{13} V_9 V_{29} &+ h_3 Q_1 d_2^c \Phi_{12} V_2^s V_{11}^s \\ + h_3 Q_1 d_2^c \Phi_{12} V_5 V_{17} &+ h_4 Q_1 d_3^c H_{15}^s H_{18}^s H_{19}^s. \end{aligned}$$

Beyond fourth order a suppression factor of $\frac{1}{10}$ per order is generally assumed. Thus, a sixth order down mass term such as these would imply $\frac{|c_j|}{n_h} \sim 1$, where j is one of $\{2, 3, 4\}$ as appropriate, when $\frac{|\bar{c}_1|}{n_{\bar{h}}} \sim 1$. However, none of our flat directions have sufficient VEVs to turn any of the Eq. (5.11) terms into mass terms.

If not sixth order, then seventh order is probably the highest order that could provide a sufficiently large bottom mass. There are no such seventh order terms containing h_1 . However, h_2 is in 15 of these terms, of which

$$h_2 Q_1 d_2^c N_3^c H_{31}^s H_{26} V_{37} \quad (5.11)$$

becomes a bottom mass term for FDN(5+8) and of which

$$h_2 Q_1 d_3^c N_2^c H_{31}^s H_{26} V_{37} \quad (5.12)$$

becomes a bottom mass term for FDN(7) (and for additional directions that lose flatness at tenth order or lower). h_3 is in 3 seventh order terms, but none of these become mass terms for any of our flat directions. h_4 appears in 17 of these terms. Φ_{12} , however, appears in 15 of the 17 and is forbidden a VEV. Φ_{13} , which also doesn't acquire a VEV in any of our directions, appears in the remaining two.

Therefore, the only possible bottom quark mass terms resulting from our set of flat directions under investigation, are the fourth order term (5.9) involving h_4 , and the seventh order terms (5.11) and (5.12) involving h_2 . Consider first the fourth order term. For this we must ask if a $\frac{|c_4|}{n_h} \gtrsim 10^{-3}$ value can be realized in our FNY model. This would require either a $h_1 \bar{h}_3$ or $h_1 \bar{h}_4$ mass term. While both terms result in a non-zero $|c_4|$, one can be shown that a $h_1 \bar{h}_3$ term gives $|c_4| \gtrsim |c_2|$, while a $h_1 \bar{h}_4$ term results in $|c_4| \lesssim |c_2|$. Therefore, we seek a $h_1 \bar{h}_3$ term. Below seventh order, the only such term is $h_1 \bar{h}_3 \Phi_{13}$. $\frac{|c_4|}{n_h} \gtrsim 10^{-3}$ would require $\langle \Phi_{13} \rangle \gtrsim 10^{-3}$ FI-scale. However, the trilinear term $\bar{\Phi}_{12} \Phi_{13} \Phi_{23}$ forbids $\bar{\Phi}_{12}$ or Φ_{13} from taking on, with Φ_{23} , a near FI-scale VEV along a stringently F -flat direction.

Thus, we must consider seventh or higher order terms that could give $h_1 \bar{h}_3$ mass terms. In Appendix B, we provide the complete set of $h_1 \bar{h}_3$ mass terms through eighth order. We find that none of our Table 1A flat directions contain appropriate VEVs to transform any of the seventh or eighth order terms into effective mass terms. While several flat directions can generate a particular ninth order mass term,

$$h_1 \bar{h}_3 \langle N_3^c \Phi_4' H_{15}^s H_{30}^s H_{31}^s H_{28} \cdot V_{37} \rangle. \quad (5.13)$$

FDN(5+8) is the only one of these that simultaneously generates the fourth order bottom mass term. FDN(5+8) has been singled out for more detailed examples and analysis in the following sections. It produces good opportunities for study of cancellations as well as phenomenologically interesting near-string-scale quark, lepton, and Higgs mass matrices. However, it must be considered as purely a training exercise since the two terms which escape all of our filters break supersymmetry at only fifth order.

5.2 Flat Direction (5+8)

The VEV set designated as FDN(5+8) is formed (cf. Table 1A of Appendix A) as the linear combination 18 FDN(5) + 1 FDN(8). More specifically, the absolute values squared of the FI-scale coefficients are combined in this ratio. Hybrid directions with desired properties may be “engineered” in this manner without disrupting Abelian D -flatness (3.4). Of course, the flaws of the constituent VEV sets can be embedded into the result as well. By using the all-order flat directions 5 and 8, this

concern is alleviated. The assigned factor of ‘18’ brings all of the non-Abelian VEVs to the same scale.

FDNA(5+8) contains fourteen fields and generates four linearly independent basis vectors for the construction of gauge invariant terms in $\langle W \rangle$ alone. In addition, each field without a VEV forms its own basis vector to be combined with the original four into dangerous F -terms. The result is a greatly expanded number of terms which can break supersymmetry, relative to the simpler directions of section (4), but along with this complexity can come a richer mass phenomenology and more interesting opportunities for cancellation. Since multiple terms may now appear within each F_{Φ_m} (cf. 3.3), one possibility which emerges is a cancellation among those components. However, this seems to require excessive tuning and to be unsustainable across many orders.

The basis combination was performed with a program which covered a coefficient parameter space large enough to produce all terms at or below order twenty five. We searched for terms with integer numbers of all fields and an even number of non-Abelian fields** with either 0 or 1 unVEVed fields (i.e. terms existing in W which FDNA(5+8) makes dangerous to W - or F -flatness). By choosing a basis set with at least one unique field per component, such computations are greatly simplified and easily guaranteed complete to a specified order. This is because the possible coefficients become restricted to a simple subset of rational numbers or, if the unique field’s strength is divided to unity, integers. Also, unless both the unique field and its vector partner take a VEV, coefficients of only one sign need be considered to find F -terms. As with the other directions in Table 1A, this occurs here only for (Φ_4) .

Our search yielded 131 dangerous terms, five of them to $\langle W \rangle$ with 11th the lowest order and 126 of them to $\langle F \rangle$, as low as order four (counting variations of (Φ_4) only once). World sheet selection rules reduced this number to 32, all of them F -terms. Disallowing more than two non-Abelian fields (foreach $SU(2)$ group) trimmed the list further to just the eight terms in Table 5.14. If a single incidence of (Φ_4) is mandated, then it is so indicated by a lack of parenthesis.

#	$O(W)$	F -term	(5.14)
1	4	$H_{16}^s \langle H_{26} \cdot V_{37} \rangle \langle N_3^c \rangle$	
2	5	$V_{32}^s \langle H_{26} \cdot V_{37} \rangle \langle \Phi_4 H_{37}^s \rangle$	
3	5	$V_{15} \langle \cdot V_{35} \rangle \langle \Phi_4' H_{30}^s H_{21}^s \rangle$	
4	5	$V_{17} \langle \cdot V_5 \rangle \langle \Phi_4' H_{30}^s H_{15}^s \rangle$	
5	8	$\Phi_{13} \langle H_{26} \cdot V_{37} \rangle \langle (\Phi_4) H_{31}^s H_{30}^s H_{15}^s N_3^c \rangle$	
6	9	$\bar{\Phi}_{13} \langle V_5 \cdot V_{35} \rangle \langle \Phi_{23} (\Phi_4)^2 H_{30}^s H_{21}^s H_{15}^s \rangle$	
7	9	$\Phi_{12} \langle H_{26} \cdot V_{37} \rangle \langle \Phi_{23} (\Phi_4) H_{31}^s H_{30}^s H_{15}^s N_3^c \rangle$	
8	10	$H_{36}^s \langle H_{26} \cdot V_{37} \rangle \langle \Phi_{23} \Phi_4 H_{31}^s H_{30}^s H_{15}^s H_{37}^s N_3^c \rangle$	

**The VEVed non-Abelian fields in FDNA(5+8) are only from $SU(2)_{H'}$.

The lowest order term (designated as #1) contains a factor of $\langle H_{26} \cdot V_{37} \rangle$ which we would like to cancel, as per the discussion of Section 3.3. This requires the VEV orientations to be chosen parallel in the three-dimensional $SU(2)_H$ adjoint space. Since FDNA(5+8) contains (two) additional non-Abelian fields with VEVs (V_5 and V_{35}) which can oppose H_{26} and V_{37} with an equal total magnitude, this choice is also D -consistent. The same factor appears in and eliminates terms #2,5,7 and 8. Since the other two non-Abelian VEVs had to be parallel as well, the contraction $\langle V_5 \cdot V_{35} \rangle$ in term #6 is also zero. In the language of [6], we could have said $s_{H_{26}} = s_{V_{37}} = 1$ and $s_{V_5} = s_{V_{35}} = -1$. This leaves us with only #3 and #4, both of which are fifth order terms with unVEVed *non-Abelian* fields so that self-cancellation is impossible. Furthermore, they will appear in different F -terms and each allows only a single (Φ_4) configuration, ruling out a couple of other (less satisfactory) scenarios. The choice $\langle \Phi'_4 \rangle = 0$, along with $\langle \bar{\Phi}'_4 \rangle = 0$ for consistency with Eqs. (3.21, 3.22), would restore F -flatness by simply removing the offending terms from $\langle F \rangle$. However, as has been discussed, this seems phenomenologically unviable and so it appears that we are stuck with a broken FDNA(5+8) at order five. As a note, the cancellations which were successful are insensitive to the factor of 18 between flat directions five and eight. Also, while it is common to see the vanishing of terms with excessive non-Abelian doublets, these mark the *only* examples wherein non-Abelian self-cancellation by selected VEVs have been found for the Table 1A flat directions.

5.3 Higgs Mass Matrices for FDNA(5+8)

Table 3 in Appendix C lists the up, down, and electron and Higgs mass matrix terms through order nine for FDNA(5+8). The Higgs terms in this table produce the mass matrix

$$M_{h_i, \bar{h}_j} = \begin{pmatrix} 0 & 0 & \lambda_9 \langle X_{13} \rangle & 0 \\ \langle \bar{\Phi}_{12} \rangle & 0 & 0 & g \langle H_{31} \rangle \\ 0 & g \langle \Phi_{23} \rangle & 0 & 0 \\ 0 & 0 & g \langle H_{38} \rangle & \lambda_5 \langle X_{44} \rangle \end{pmatrix} \quad (5.15)$$

$$\text{where } X_{13} = \frac{N_3^c \Phi'_4 H_{15}^s H_{30}^s H_{31}^s H_{28} \cdot V_{37}}{M_{str}^6} \text{ and } X_{44} = \frac{\phi_{23} H_{31}^s H_{38}^s}{M_{str}^2}.$$

In [6, 7] we showed that a small $\bar{\Phi}_{12} \ll$ FI-scale VEV produces superior quark and lepton mass matrix phenomenology. Thus, in the matrix (5.15) we allow possible contributions from the mass term $h_2 \bar{h}_1 \langle \bar{\Phi}_{12} \rangle$. However, since $\bar{\Phi}_{12}$ does not acquire a VEV in any of our flat directions in Table 1A, $\langle \bar{\Phi}_{12} \rangle \neq 0$ would have to arise as a second order effect at or below the GUT scale.

When we allow $\langle \bar{\Phi}_{12} \rangle \sim 10^{-4}$, the numeric form of the FDNA(5+8) Higgs doublet

mass matrix is^{††}

$$M_{h_i, \bar{h}_j} \sim \begin{pmatrix} 0 & 0 & 10^{-5} & 0 \\ 10^{-4} & 0 & 0 & 1 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 10^{-1} \end{pmatrix}. \quad (5.16)$$

Using (5.5), the near-massless Higgs eigenstates produced from Eq. (5.16) are:

$$h = h_1 + 10^{-7}h_2 - 10^{-5}h_4 \quad (5.17)$$

$$\bar{h} = \bar{h}_1 + 10^{-6}\bar{h}_3 - 10^{-4}\bar{h}_4. \quad (5.18)$$

Note that the coefficient of \bar{h}_4 is approximately $\bar{\Phi}_{12}$. In the limit of $\langle \bar{\Phi}_{12} \rangle = 0$, the h eigenstate reduces to \bar{h}_1 .

These Higgs eigenstates provide examples of how several orders of magnitude mass suppression factors can appear in the low order terms of a Minimal Standard Heterotic-String Model. Here, specifically, the h_4 coefficient in (5.17) can provide 10^{-5} mass suppression for one down-quark generation and one electron generation, since $h_4(Q_i d_j^c + L_i e_j^c)$, when rewritten in terms of the Higgs mass eigenstates, contains a factor of $10^{-5}h(Q_i d_j^c + L_i e_j^c)$. Similarly, the h_2 coefficient can provide 10^{-7} suppression. Further, the \bar{h}_4 coefficient in (5.18) can provide 10^{-4} up like-quark mass suppression and the \bar{h}_3 coefficient a corresponding 10^{-6} suppression. We examine the quark and charged-lepton eigenstates in the following subsection.

5.4 Quark and Lepton Mass Matrices for FDNA(5+8)

The matrices in this section are calculated up to ninth order, where the suppression in the coupling (assumed here to be $\sim 10^{-5}$) is comparable to that coming out of Eqs. (5.17, 5.18). The terms comprising these matrices (cf. Appendix C, Table 3) take the form $\bar{h}_i Q_j u_k^c$, $h_i Q_j d_k^c$, and $h_i L_j e_k^c$, plus possibly some number of VEVed fields. Not having an exact value for the couplings, we will also not concern ourselves here with the specific magnitudes of the VEVs or overall phases for the massterms.

The up-quark mass matrix for FDNA(5+8) contains only a single term (corresponding to the top mass) when $\langle \bar{\Phi}_{12} \rangle = 0$, but develops an interesting texture when $\langle \bar{\Phi}_{12} \rangle \sim 10^{-4}$ FI-scale. To leading order, the general form is

$$M_{Q, u^c} = \begin{pmatrix} \bar{h}_1 & .1\bar{h}_3 + 10^{-3}\bar{h}_4 & 0 \\ 10^{-2}\bar{h}_3 + 10^{-4}\bar{h}_4 & 0 & 0 \\ 0 & 0 & \bar{h}_4 \end{pmatrix}.$$

$$\sim \begin{pmatrix} 1 & 10^{-6} & 0 \\ 10^{-7} & 0 & 0 \\ 0 & 0 & 10^{-4} \end{pmatrix}. \quad (5.19)$$

^{††}As stated in Section 5.1, we will assume a suppression in the superpotential coupling of $\frac{1}{10}$ per order above fourth.

Just as for the Higgs, discussed in the start of Section 5.1, the $m_{u^c}^2$ are obtained by diagonalizing $M^\dagger M$. In top quark mass units, the up-like mass eigenvalues are 1, 10^{-4} , and 10^{-13} . Clearly, a more realistic structure would appear if the \bar{h}_4 suppression factor was 10^{-2} rather than 10^{-4} .

The FDNA(5+8) down-quark mass matrix is independent of the $\bar{\Phi}_{12}$ VEV value. It has the form

$$\begin{aligned} M_{Q,d^c} &= \begin{pmatrix} 0 & 10^{-3}h_2 + 10^{-5}h_4 & h_4 \\ 10^{-5}h_2 & h_2 + .1h_4 & 10^{-5}h_2 \\ 0 & 10^{-4}h_2 & 0 \end{pmatrix} \\ &\sim \begin{pmatrix} 0 & 10^{-9} & 10^{-5} \\ 10^{-11} & 10^{-6} & 10^{-11} \\ 0 & 10^{-10} & 0 \end{pmatrix}. \end{aligned} \quad (5.20)$$

The resulting down-quark mass eigenvalues are 10^{-5} , 10^{-6} , and 10^{-15} (again in top quark mass units). One might suggest this provides quasi-realistic down and strange masses, but lacks a bottom mass. Unfortunately, the down-like quark eigenstate corresponding to a 10^{-5} mass is in Q_1 , arising with its suppression factor from the term $h_4 Q_1 d_3^c H_{21}^s$. From our discussion above, this is in fact the bottom quark. The second and third generation masses would be more viable if the h_4 suppression factor were 10^{-2} instead.

The FDNA(5+8) charged-lepton mass matrix (likewise independent of the $\bar{\Phi}_{12}$ VEV value) takes the form

$$\begin{aligned} M_{L,e^c} &= \begin{pmatrix} 0 & 10^{-4}h_2 & 0 \\ 10^{-4}h_2 & h_2 + .1h_4 & 10^{-4}h_2 \\ .1h_4 & 10^{-5}h_2 & 0 \end{pmatrix} \\ &\sim \begin{pmatrix} 0 & 10^{-10} & 0 \\ 10^{-10} & 10^{-6} & 10^{-10} \\ 10^{-5} & 10^{-11} & 0 \end{pmatrix}. \end{aligned} \quad (5.21)$$

The three corresponding electron-like mass eigenvalues 10^{-5} , 10^{-6} , and 10^{-14} . As with the down-like quark masses, more viable second and third generation electron masses would appear if the h_4 suppression factor was only 10^{-2} .

For both M_{Q,d^c} and M_{L,e^c} , the (2, 2) element is composed by two terms of similar magnitude. The mass phenomenology degrades further if these contributions cancel significantly. Also, the cancellations considered in Section 5.2 are not without an effect on these mass ratios. If non-Abelian self-cancellation is implemented, then the ninth order contribution to M_{h_i, \bar{h}_j} , Eq. (5.15), vanishes and the massless Higgs becomes simply h_1 . Since neither d^c nor e^c appear to this order in mass terms with h_1 , their mass hierarchies also vanish. \bar{h} is insensitive to this particular change, as are the leading order terms of M_{Q,u^c} , Eq (5.19). If $\langle \Phi'_4 \rangle = \langle \bar{\Phi}'_4 \rangle = 0$ were further enforced, the only effect here would be the loss of the lightest up-like quark.

6 Concluding Remarks

The more realistic free fermionic string models are the most realistic string models constructed to date. By expanding our flat direction search to allow VEVs to non-Abelian charged fields,[7] rather than limiting VEVs to non-Abelian singlets,[4, 5, 6] we have improved the phenomenology of our Minimal Standard Heterotic String Model. We have found that quasi-realistic patterns to quark and charged-lepton mass matrices can appear. The improved phenomenology was a result of both the non-Abelian VEVs and the structure of the physical Higgs doublets h and \bar{h} . In the more realistic free fermionic heterotic models, each of these Higgs can contain up to four components with vastly different weights, differing by several orders of magnitude. These components generically all couple differently to a given $Q_i u_j^c$, $Q_i d_j^c$, or $L_i e_j^c$. Thus, mass suppression factors for the first and second quark and lepton generations can appear even at very low order as a result of the different weights of the Higgs components. In this model we found that, while the top quark can receive a viable, unsuppressed mass (given realistic Higgs VEVs), the bottom quark mass, most second generation and some first generation masses were too small. This was, a result of the weight factors for one h component and for one \bar{h} component, being too low. Phenomenology would have improved significantly if the h_4 and \bar{h}_4 weights in h and \bar{h} , respectively, were larger by a factor of 100 than their values of 10^{-5} and 10^{-4} found for our best non-Abelian flat direction, FDNA(5+8). Also, we have observed the emergence of new techniques for the removal of dangerous terms from $\langle W \rangle$ and from $\langle F \rangle$. In Table 1B, four of our flat directions are lifted to all order by the vanishing of terms with more than two non-Abelian fields. Non-Abelian self-cancellation within single terms is another promising tool for extending the order to which a direction is safe.

The existence of Minimal Standard Heterotic String Models, which contain solely the three generations of MSSM quarks and leptons and a pair of Higgs doublets as the massless SM-charged states in the low energy effective field theory, has been a significant discovery. Our FNY model has been the first example MSHSM. Clearly, though the stringently flat F - and D -flat directions that we have found can produce this MSHSM, they do not themselves lead to viable quark and lepton mass matrices. Nevertheless, we have found that these flat directions can present some interesting phenomenological features such as multi-component physical Higgs that couple differently to given quarks and leptons. One direction suggested by the partial success of these flat directions is investigation of non-stringently flat directions for the FNY model that are flat to a finite order due to cancellation between various components in an F -term. The necessity of non-stringent F -flatness was recently also shown for free fermionic flipped $SU(5)$ models [21]. Further, our discovery of an MSHSM in the neighborhood of the string/M-theory parameter space allowing free-fermionic description strongly suggests a search for further, perhaps more phenomenologically realistic, MSHSMs in this region. This we leave for future research.

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A Dimension-1 Non-Abelian D -flat MSSM Directions

FD#	$\mathcal{O}(W)$	Q'	$\Phi_{23}(\Phi_4)H_{38,31}^s$	$\bar{\Phi}_{56}\Phi'_{56}H_{30,21,15,17,36,37}^s$	$N_{i=1,2,3}^c$	$H_{28}V_{40}$	$H_{23,26}V_{5,7,35,37}$
5	∞	-1	2 2 1 1	0 0 4 0 3 0 0 0	0 0 1	00	01 001 0
6	∞	-1	3 3 1 1	0 0 5 0 3 0 0 0	1 1 0	00	02 002 0
7	∞	-2	7 10 2 4	1 0 12 0 8 0 0 0	6 0 0	00	06 000 6
8	∞	-2	9 22 2 4	0 -1 16 8 0 0 0 10	0 0 0	00	00180018
9	∞	-2	9 6 2 4	0 -1 16 0 0 80 10	0 0 0	00	00100010
10	10	-2	6 4 2 2	0 0 8 0 4 0 0 0	0 4 0	00	04 002 2
11	9	-1	4 2 1 1	0 -1 4 0 0 0 0 0	0 4 0	00	04 001 3
12	9	-2	8 -4 2 4	1 -1 6 0 0 0 0 0	0 8 0	88	00 000 0
13	8	-2	8 4 2 4	1 -1 6 0 0 0 0 0	0 8 0	00	08 000 8
14	8	-1	7 5 4 8	5 1 0 0 0 0 0 0	0 4 0	00	37 000 10
15	8	-1	5 3 2 4	2 0 2 0 0 0 0 0	0 4 0	00	15 000 6
16	8	-2	7 4 2 4	1 0 6 0 2 0 0 0	0 6 0	00	06 000 6
17	7	-4	15 6 2 4	0 -3 14 0 0 0 0 0	0 16 0	00	016 020 14
18	7	-2	7 5 3 1	0 0 10 0 6 0 1 0	0 2 0	00	04 004 0
5+8	5	-20	45 58 20 22	0 -1 88 8 54 0 0 10	0018	00	018 180 1818

Table 1A: FNY directions flat through at least seventh order that contain VEVs of Non-Abelian charged Hidden Sector Fields. All component VEVs in these directions are uncharged under the MSSM gauge group. Column one entries specify the class to which an example direction belongs. Column two entries give the anomalous charges $Q' \equiv Q^{(A)}/112$ of the flat directions. The next several column entries specify the ratios of the norms of the VEV²s. The Φ_4 -related component is the net value (in units of the square overall VEV scale) of $|\langle\Phi_4\rangle|^2 + |\langle\Phi'_4\rangle|^2 - |\langle\bar{\Phi}_4\rangle|^2 - |\langle\bar{\Phi}'_4\rangle|^2$. E.g., a “1” in the Φ_4 column for FDNA1 specifies that $|\langle\Phi_4\rangle|^2 + |\langle\Phi'_4\rangle|^2 - |\langle\bar{\Phi}_4\rangle|^2 - |\langle\bar{\Phi}'_4\rangle|^2 = 1 \times |\langle\alpha\rangle|^2$. For consistency, we continue here the notation of [7]. However, since FDNA1 through FDNA4 contain the unwanted VEV $\langle\Phi_{12}\rangle$, they are excluded and we begin with FDNA5.

FD#	$\mathcal{O}(W)$	Dangerous W Terms	Comments
5	∞	None	No gauge invariant $\langle F \rangle$ terms
6	∞	None	13 th order NAFE
7	∞	None	13 th order NAFE
8	∞	None	Safe by string selection rules
9	∞	None	Safe by string selection rules
10	∞	None	10 th and 12 th order NAFE
11	9	$\langle \Phi_{23}(\Phi_4) H_{38}^s \bar{\Phi}_{56} H_{30}^s N_2^c H_{26} \cdot V_{35} \rangle H_{20}^s$	No NASC
12	9	$\langle \Phi_{23}(\bar{\Phi}_4) H_{38}^s H_{31}^s \bar{\Phi}_{56} N_2^c H_{28} \cdot V_{40} \rangle H_{22}^s$	No NASC
13,14,15,16	8	$\langle \Phi_{23} H_{38}^s H_{31}^s \bar{\Phi}_{56} N_2^c H_{26} \cdot V_{37} \rangle H_{22}^s$	No NASC
17	∞	None	7 th and 12 th order NAFE
18	7	$\langle \Phi_{23} H_{30}^s H_{15}^s H_{36} N_2^c H_{26} \cdot \rangle V_7$	No NASC
5+8	5	$\langle \Phi_4' H_{30}^s H_{21}^s V_{35} \cdot \rangle V_{15}$	No NASC
	5	$\langle \Phi_4' H_{30}^s H_{15}^s V_5 \cdot \rangle V_{17}$	No NASC

Table 1B: All superpotential terms which break F -flatness in the Table 1A directions. Column one entries specify the class of a flat direction. The entry in the next column specifies superpotential terms which break F -flatness. For the final column, the notation “NAFE” indicates that the direction was made safe by the removal of terms at the specified order(s) with a non-Abelian field excess (always four total for these cases). “No NASC” means that D -constraints would not permit a consistent non-Abelian self-cancellation solution. For FDNA(5+8), which breaks at order five, there are self-cancellations as low as order four. For details, see Table 5.14 and the surrounding discussion of Section 5.2.

B Potential FNY Quark and Charged Lepton Mass Terms (through 8th order)

Possible up-quark mass terms from $\langle \bar{h}_1 \rangle$:

$$\begin{aligned}
& \bar{h}_1 [Q_1 u_1^c \\
& + Q_1 u_2^c H_{15}^s H_{30}^s V_{11}^s V_{12}^s \\
& + Q_1 u_2^c H_{15}^s H_{30}^s V_{15}^s V_{17}^s \\
& + Q_1 u_3^c H_{30}^s H_{38}^s V_{15}^s V_{35}^s \\
& + Q_3 u_1^c H_{17}^s H_{30}^s V_{15}^s V_{27}^s \\
& + Q_1 u_2^c \Phi_{23} H_{15}^s H_{30}^s H_{36}^s H_{37}^s \\
& + Q_1 u_2^c \Phi_{23} H_{15}^s H_{30}^s V_{29}^s V_{30}^s \\
& + Q_1 u_2^c \bar{\Phi}_{56} H_{32}^s H_{38}^s V_{21}^s V_{32}^s \\
& + Q_1 u_3^c N_2^c \bar{\Phi}_4 H_{30}^s H_{36}^s V_{31}^s \\
& + Q_1 u_3^c \bar{\Phi}_4 H_{17}^s H_{30}^s V_{15}^s V_{27}^s \\
& + Q_1 u_3^c \bar{\Phi}_{56} H_{23} H_{26} V_{17} V_{37} \\
& + Q_2 u_1^c N_3^c \bar{\Phi}_{56} H_{30}^s H_{36}^s V_{31}^s \\
& + Q_2 u_1^c \bar{\Phi}_4 H_{15}^s H_{30}^s V_{14} V_{13} \\
& + Q_2 u_1^c \bar{\Phi}_{56} H_{16}^s H_{32}^s H_{38}^s H_{39}^s \\
& + Q_2 u_1^c \bar{\Phi}_{56} H_{30}^s V_{31}^s H_{42} V_{23} \\
& + Q_3 u_1^c N_2^c \bar{\Phi}_{13} H_{30}^s H_{28} V_{40} \\
& + Q_3 u_1^c \bar{\Phi}_4 H_{15}^s H_{30}^s V_{14} V_{23} \\
& + Q_3 u_1^c \bar{\Phi}_{56} H_{17}^s H_{30}^s V_{20} V_{29} \\
& + Q_3 u_1^c \bar{\Phi}_{56} H_{25} V_{40} H_{26} V_{17} \\
& + Q_3 u_3^c \bar{\Phi}_{12} H_{19}^s H_{30}^s V_{15} V_{37} \\
& + \dots] \\
& + Q_1 u_2^c H_{15}^s H_{30}^s V_{14} V_{13} \\
& + Q_1 u_2^c H_{15}^s H_{30}^s V_{11}^s V_{22}^s \\
& + Q_3 u_1^c N_2^c H_{30}^s H_{36}^s V_{31}^s \\
& + Q_3 u_1^c H_{17}^s H_{30}^s V_{19} V_{30}^s \\
& + Q_3 u_3^c V_{30} V_{40} V_{27} V_{37} \\
& + Q_1 u_2^c H_{15}^s H_{30}^s V_{19} V_{20} \\
& + Q_1 u_3^c H_{15}^s H_{30}^s V_{14} V_{23} \\
& + Q_3 u_1^c H_{17}^s H_{30}^s V_{19} V_{30} \\
& + Q_3 u_3^c V_{30} V_{40} V_{27} V_{37} \\
& + Q_1 u_2^c \Phi_{23} H_{15}^s H_{30}^s V_{24} V_{23} \\
& + Q_1 u_2^c \Phi_{23} H_{30}^s H_{37}^s V_{25} V_{37} \\
& + Q_1 u_3^c N_2^c \bar{\Phi}_{13} H_{30}^s H_{26} V_{37} \\
& + Q_1 u_3^c \bar{\Phi}_4 H_{30}^s H_{38}^s V_{19} V_{39} \\
& + Q_1 u_3^c \bar{\Phi}_{56} H_{25} V_{20} H_{26} V_{37} \\
& + Q_1 u_3^c \bar{\Phi}_{56} H_{15}^s H_{30}^s V_{12}^s V_{21}^s \\
& + Q_2 u_1^c \Phi_{23} H_{30}^s H_{37}^s V_{29} V_{40} \\
& + Q_2 u_1^c \bar{\Phi}_4 H_{15}^s H_{30}^s V_{19} V_{20} \\
& + Q_2 u_1^c \bar{\Phi}_{56} H_{16}^s H_{38}^s H_{42} H_{35} \\
& + Q_2 u_2^c \Phi_{23} \bar{\Phi}_{56} \bar{\Phi}_{56} H_{36}^s H_{38}^s \\
& + Q_3 u_1^c \bar{\Phi}_{23} H_{30}^s H_{39}^s V_{12}^s V_{31}^s \\
& + Q_3 u_1^c \bar{\Phi}_4 H_{15}^s H_{30}^s V_{11}^s V_{22}^s \\
& + Q_3 u_1^c \bar{\Phi}_{56} H_{17}^s H_{30}^s V_{17} V_{25} \\
& + Q_3 u_1^c \bar{\Phi}_{56} H_{28} V_{40} H_{23} V_{17} \\
& + Q_3 u_3^c \bar{\Phi}_4 V_{30} V_{37} V_{37} \\
& + Q_3 u_3^c \bar{\Phi}_4 H_{18}^s H_{38}^s V_{19} V_{40} \\
& + Q_3 u_1^c \bar{\Phi}_{23} H_{30}^s V_{31}^s H_{42} V_{13} \\
& + Q_3 u_1^c \bar{\Phi}_4 H_{30}^s H_{38}^s V_{15} V_{35} \\
& + Q_3 u_1^c \bar{\Phi}_{56} H_{25} H_{28} V_{20} V_{40} \\
& + Q_3 u_2^c \bar{\Phi}_{56} H_{18}^s H_{38}^s V_{19} V_{40} \\
& + Q_3 u_3^c \bar{\Phi}_4 V_{40} V_{27} V_{27}
\end{aligned}$$

Possible up-quark mass terms from $\langle \bar{h}_2 \rangle$:

$$\begin{aligned}
& \bar{h}_2 [Q_1 u_1^c H_{29}^s H_{30}^s V_{31}^s V_{32}^s \\
& + Q_1 u_1^c N_1^c \Phi_{12} H_{30}^s H_{32}^s V_1^s \\
& + Q_1 u_1^c N_3^c \Phi_{12} H_{30}^s V_{24} H_{35} \\
& + Q_1 u_1^c \Phi_{12} H_{17}^s H_{30}^s V_9 V_{19} \\
& + Q_1 u_1^c \Phi_{12} H_{19}^s V_{12} V_{34} H_{35} \\
& + Q_1 u_1^c \Phi_{12} H_{30}^s H_{37}^s H_{42} H_{35} \\
& + Q_1 u_3^c N_2^c \Phi_{23} H_{30}^s H_{26} V_{37} \\
& + Q_1 u_3^c \Phi_{12} H_{30}^s H_{38}^s V_{15} V_{35} \\
& + Q_1 u_3^c \bar{\Phi}_{56} H_{21}^s H_{30}^s H_{42} H_{35} \\
& + Q_3 u_1^c N_2^c \Phi_{12} H_{30}^s H_{36}^s V_{31} \\
& + Q_3 u_1^c \Phi_{12} H_{17}^s H_{30}^s V_{15} V_{27} \\
& + Q_3 u_1^c \Phi_{13} H_{30}^s H_{39}^s V_{12} V_{31} \\
& + Q_3 u_3^c \bar{\Phi}_{56} H_{30}^s H_{32}^s H_{38}^s H_{39}^s \\
& + Q_1 u_2^c H_{15}^s H_{30}^s V_{31}^s V_{32}^s \\
& + Q_1 u_1^c N_3^c \Phi_{12} H_{16}^s H_{26} V_{37} \\
& + Q_1 u_1^c \Phi_{12} H_{15}^s H_{30}^s V_{14} V_3 \\
& + Q_1 u_1^c \Phi_{12} H_{19}^s H_{31}^s V_{15} V_{37} \\
& + Q_1 u_1^c \Phi_{12} H_{21}^s H_{30}^s V_{19} V_{39} \\
& + Q_1 u_1^c \Phi_{12} H_{30}^s V_{31}^s H_{23} V_{35} \\
& + Q_1 u_3^c \Phi_{12} H_{15}^s H_{30}^s V_{11}^s V_{22} \\
& + Q_1 u_3^c \Phi_{12} V_9 V_{40} V_{27} V_{37} \\
& + Q_2 u_1^c \Phi_{12} H_{16}^s H_{19}^s V_{15} V_{37} \\
& + Q_3 u_1^c N_2^c \Phi_{23} H_{30}^s H_{28} V_{40} \\
& + Q_3 u_1^c \Phi_{12} H_{30}^s H_{38}^s V_{19} V_{39} \\
& + Q_3 u_1^c \Phi_{13} H_{30}^s V_{31}^s H_{42} V_{13} \\
& + Q_3 u_3^c \bar{\Phi}_{4} H_{19}^s H_{30}^s V_{19} V_{40} \\
& + Q_3 u_3^c \bar{\Phi}_{56} H_{38}^s V_{32}^s H_{28} V_{40} \\
& + \dots]
\end{aligned}$$

Possible up-quark mass terms from $\langle \bar{h}_3 \rangle$:

$$\begin{aligned}
& \bar{h}_3 [Q_1 u_1^c H_{29}^s H_{30}^s + Q_2 u_2^c \Phi_4' H_{15}^s H_{30}^s \\
& + Q_1 u_3^c N_2^c H_{30}^s H_{26} V_{37} + Q_1 u_1^c N_1^c \Phi_{13} H_{30}^s H_{32}^s V_1 \\
& + Q_1 u_1^c N_3^c \Phi_{13} H_{30}^s V_{24} H_{35} + Q_1 u_1^c \Phi_{13} H_{17}^s H_{30}^s V_9 V_{19} \\
& + Q_1 u_1^c \Phi_{13} H_{19}^s V_{12} V_{34} H_{35} + Q_1 u_1^c \Phi_{13} H_{30}^s H_{37}^s H_{42} H_{35} \\
& + Q_1 u_2^c N_3^c \bar{\Phi}_{56}^s H_{30}^s H_{26} V_{37} + Q_1 u_3^c \Phi_{13} H_{15}^s H_{30}^s V_{11} V_{22} \\
& + Q_1 u_3^c \Phi_{13} V_9 V_{40} V_{27} V_{37} + Q_2 u_1^c \Phi_{12} H_{30}^s H_{37}^s V_{25} V_{37} \\
& + Q_2 u_3^c \bar{\Phi}_{56}^s H_{15}^s H_{22}^s H_{30}^s H_{38}^s + Q_3 u_1^c N_2^c \Phi_4' H_{30}^s H_{26} V_{37} \\
& + Q_3 u_1^c \Phi_{13} H_{30}^s H_{38}^s V_{19} V_{39} + Q_3 u_2^c \bar{\Phi}_{56}^s H_{15}^s H_{22}^s H_{30}^s H_{38}^s \\
& + Q_3 u_3^c \bar{\Phi}_{56}^s H_{21}^s H_{30}^s H_{29}^s H_{36}^s + Q_1 u_1^c H_{30}^s H_{32}^s V_{21} \\
& + Q_1 u_1^c \Phi_{13} H_{17}^s H_{32}^s V_{14} V_{13} + Q_1 u_1^c \Phi_{13} H_{19}^s H_{32}^s V_{34} V_{13} \\
& + Q_1 u_1^c \Phi_{13} H_{30}^s H_{32}^s H_{37}^s H_{39}^s + Q_1 u_1^c \Phi_{13} H_{37}^s V_{32}^s H_{28} V_{40} \\
& + Q_1 u_2^c N_3^c \bar{\Phi}_4^s H_{30}^s H_{28} V_{40} + Q_1 u_3^c \Phi_{13} H_{30}^s H_{38}^s V_{15} V_{35} \\
& + Q_2 u_1^c N_3^c \bar{\Phi}_{56}^s H_{30}^s H_{28} V_{40} + Q_2 u_2^c \bar{\Phi}_{56}^s H_{19}^s H_{30}^s V_{15} V_{37} \\
& + Q_2 u_3^c \bar{\Phi}_{56}^s H_{21}^s H_{30}^s V_{29} V_{40} + Q_3 u_1^c \Phi_{13} H_{17}^s H_{30}^s V_{15} V_{27} \\
& + Q_3 u_1^c \bar{\Phi}_{56}^s H_{22}^s H_{29}^s H_{30}^s H_{38}^s + Q_3 u_3^c \bar{\Phi}_{23}^s H_{19}^s H_{30}^s V_{15} V_{37} \\
& + \dots]
\end{aligned}$$

Possible up-quark mass terms from $\langle \bar{h}_4 \equiv H_{34} \rangle$:

$$\begin{aligned}
& h_4 [Q_3 u_3^c H_{30}^s \\
& + Q_1 u_1^c \Phi_{12} H_{31}^s \\
& + Q_1 u_2^c \Phi_{12} \overline{\Phi}_4' H_{16}^s \\
& + Q_1 u_1^c \Phi_{13} H_{31}^s V_{31}^s V_{32}^s \\
& + Q_1 u_3^c \Phi_{12} H_{30}^s V_2^s V_{21}^s \\
& + Q_2 u_1^c \Phi_{23} H_{16}^s H_{29}^s H_{30}^s \\
& + Q_2 u_3^c \Phi_{12} H_{30}^s V_{12}^s V_{21}^s \\
& + Q_3 u_1^c \Phi_{12} H_{30}^s V_{24}^s V_3 \\
& + Q_3 u_2^c \Phi_{12} H_{30}^s V_{24}^s V_{13} \\
& + Q_1 u_1^c \Phi_{12} \Phi_{13} H_{16}^s V_2^s V_{11}^s \\
& + Q_1 u_1^c \Phi_{12} \Phi_{13} H_{18}^s V_7 V_{15} \\
& + Q_1 u_2^c \Phi_{12} \overline{\Phi}_{56}' H_{30}^s V_5 V_{17} \\
& + Q_1 u_2^c \Phi_{13} \overline{\Phi}_4' H_{16}^s V_{31}^s V_{32}^s \\
& + Q_1 u_3^c \Phi_{12} \Phi_{13} H_{18}^s V_{19} V_{30} \\
& + Q_1 u_3^c \Phi_{12} \overline{\Phi}_4' H_{30}^s V_{24}^s V_3 \\
& + Q_1 u_3^c \Phi_{13} \Phi_{23} H_{30}^s V_2^s V_{21}^s \\
& + Q_2 u_1^c \Phi_{12} \overline{\Phi}_{56}' H_{30}^s V_{14}^s V_3 \\
& + Q_2 u_2^c H_{15}^s H_{16}^s H_{30}^s V_{31}^s V_{32}^s \\
& + Q_2 u_3^c \Phi_{12} \overline{\Phi}_{56}' H_{30}^s V_{14}^s V_{23} \\
& + Q_3 u_1^c \Phi_{12} \Phi_4' H_{30}^s V_2^s V_{21}^s \\
& + Q_3 u_1^c \Phi_{13} \Phi_{23} H_{22}^s H_{30}^s H_{37}^s \\
& + Q_3 u_2^c \Phi_{12} \overline{\Phi}_{56}' H_{16}^s H_{21}^s H_{36}^s \\
& + \dots]
\end{aligned}
\begin{aligned}
& + Q_2 u_1^c \Phi_{12} H_{16}^s \\
& + Q_1 u_1^c \Phi_{23} H_{29}^s H_{30}^s H_{31}^s \\
& + Q_1 u_3^c \Phi_{12} H_{30}^s V_9 V_{29} \\
& + Q_2 u_2^c \Phi_{23} H_{15}^s H_{16}^s H_{30}^s \\
& + Q_2 u_3^c \Phi_{12} H_{30}^s V_{24} V_{13} \\
& + Q_3 u_1^c \Phi_{12} H_{30}^s V_7 V_{25} \\
& + Q_1 u_1^c \Phi_{12} \Phi_{13} H_{16}^s V_5 V_{17} \\
& + Q_1 u_1^c H_{29}^s H_{30}^s H_{31}^s V_{31}^s V_{32}^s \\
& + Q_1 u_2^c \Phi_{12} \overline{\Phi}_{56}' H_{32}^s V_{28} V_{30} \\
& + Q_1 u_2^c \Phi_{23} \overline{\Phi}_4' H_{16}^s H_{29}^s H_{30}^s \\
& + Q_1 u_3^c \Phi_{12} \Phi_{13} H_{18}^s V_{15} V_{27} \\
& + Q_1 u_3^c \Phi_{12} \overline{\Phi}_4' H_{30}^s V_7 V_{25} \\
& + Q_1 u_3^c \Phi_{13} \Phi_{23} H_{30}^s V_9 V_{29} \\
& + Q_2 u_1^c \Phi_{23} \Phi_4' H_{15}^s H_{30}^s H_{31}^s \\
& + Q_2 u_3^c \Phi_{12} \overline{\Phi}_{56}' H_{16}^s H_{21}^s H_{36}^s \\
& + Q_3 u_1^c \Phi_{12} \Phi_{13} H_{16}^s V_{11}^s V_{22}^s \\
& + Q_3 u_1^c \Phi_{12} \Phi_4' H_{30}^s V_9 V_{29} \\
& + Q_3 u_1^c \Phi_{13} \Phi_{23} H_{30}^s V_{24} V_3 \\
& + Q_3 u_2^c \Phi_{12} \overline{\Phi}_{56}' H_{30}^s V_{11}^s V_{22}^s \\
& + Q_1 u_1^c \Phi_{12} \Phi_{13} H_{16}^s V_{18}^s V_{12}^s \\
& + Q_1 u_2^c \Phi_{12} \overline{\Phi}_{56}' H_{30}^s V_2^s V_{11}^s \\
& + Q_1 u_2^c \Phi_{12} \overline{\Phi}_{56}' V_{32}^s H_{26} V_{27} \\
& + Q_1 u_2^c H_{15}^s H_{30}^s H_{31}^s V_{31}^s V_{32}^s \\
& + Q_1 u_3^c \Phi_{12} \overline{\Phi}_4' H_{22}^s H_{30}^s H_{37}^s \\
& + Q_1 u_3^c \Phi_{12} \overline{\Phi}_{56}' H_{21}^s H_{31}^s H_{36}^s \\
& + Q_1 u_3^c H_{15}^s H_{30}^s V_{32}^s H_{26} V_7 \\
& + Q_2 u_1^c H_{16}^s H_{29}^s H_{30}^s V_{31}^s V_{32}^s \\
& + Q_2 u_3^c \Phi_{12} \overline{\Phi}_{56}' H_{30}^s V_{11}^s V_{22}^s \\
& + Q_3 u_1^c \Phi_{12} \Phi_{13} H_{16}^s V_{14} V_{23} \\
& + Q_3 u_1^c \Phi_{12} \overline{\Phi}_{56}' H_{22}^s H_{31}^s H_{38}^s \\
& + Q_3 u_1^c \Phi_{13} \Phi_{23} H_{30}^s V_7 V_{25} \\
& + Q_3 u_2^c \Phi_{12} \overline{\Phi}_{56}' H_{30}^s V_{14} V_{23}
\end{aligned}$$

Possible down–quark mass terms from $\langle h_1 \rangle$:

$$\begin{aligned}
& h_1 [Q_3 d_3^c H_{29}^s H_{30}^s \\
& + Q_2 d_2^c H_{29}^s H_{30}^s V_{31}^s V_{32}^s \\
& + Q_1 d_2^c N_3^c \Phi_{12} H_{31}^s H_{26}^s V_{37} \\
& + Q_1 d_2^c \Phi_{12} H_{29}^s V_{32}^s H_{28}^s V_{30} \\
& + Q_1 d_3^c \Phi_{12} H_{29}^s H_{30}^s V_2^s V_{21} \\
& + Q_2 d_1^c \Phi_{12} H_{29}^s H_{30}^s V_1^s V_{12} \\
& + Q_2 d_2^c N_3^c \Phi_{12} H_{16}^s H_{26}^s V_{37} \\
& + Q_2 d_2^c \Phi_{12} H_{15}^s H_{30}^s V_{14}^s V_3 \\
& + Q_2 d_2^c \Phi_{12} H_{19}^s H_{31}^s V_{15}^s V_{37} \\
& + Q_2 d_2^c \Phi_{12} H_{21}^s H_{30}^s V_{19}^s V_{39} \\
& + Q_2 d_2^c \Phi_{12} H_{30}^s V_{31}^s H_{23}^s V_{35} \\
& + Q_2 d_3^c N_2^c \Phi_{12} H_{30}^s H_{32}^s V_{21}^s \\
& + Q_2 d_3^c \Phi_{12} H_{15}^s H_{30}^s V_{24}^s V_3 \\
& + Q_2 d_3^c \Phi_{12} H_{29}^s H_{30}^s V_{24}^s V_{13} \\
& + Q_3 d_1^c \Phi_{12} H_{29}^s H_{30}^s V_1^s V_{22} \\
& + Q_3 d_2^c \Phi_{12} H_{17}^s H_{30}^s V_4^s V_{23} \\
& + Q_3 d_2^c \Phi_{12} H_{19}^s H_{32}^s V_{34}^s V_{23} \\
& + Q_3 d_2^c \Phi_{12} H_{29}^s H_{30}^s V_{14}^s V_{23} \\
& + Q_3 d_3^c N_1^c \Phi_{13} H_{30}^s H_{32}^s V_1^s \\
& + Q_3 d_3^c N_3^c \Phi_{13} H_{30}^s V_{24}^s H_{35} \\
& + Q_3 d_3^c \Phi_{13} H_{17}^s H_{30}^s V_9^s V_{19} \\
& + Q_3 d_3^c \Phi_{13} H_{19}^s V_{12}^s V_{34}^s H_{35} \\
& + Q_3 d_3^c \Phi_{13} H_{30}^s H_{37}^s H_{42}^s H_{35} \\
& + Q_1 d_2^c \Phi_{12} H_{29}^s H_{30}^s V_2^s V_{11} \\
& + Q_1 d_2^c \Phi_{12} H_{29}^s V_{32}^s H_{26}^s V_{27} \\
& + Q_1 d_3^c \Phi_{12} H_{29}^s H_{30}^s V_9^s V_{29} \\
& + Q_2 d_1^c \Phi_{12} H_{29}^s H_{30}^s V_7^s V_{15} \\
& + Q_2 d_2^c N_3^c \Phi_{12} H_{30}^s H_{32}^s V_{21}^s \\
& + Q_2 d_2^c \Phi_{12} H_{17}^s H_{30}^s V_4^s V_{13} \\
& + Q_2 d_2^c \Phi_{12} H_{19}^s H_{32}^s V_{34}^s V_{13} \\
& + Q_2 d_2^c \Phi_{12} H_{30}^s H_{32}^s H_{37}^s H_{39}^s \\
& + Q_2 d_2^c \Phi_{12} H_{37}^s V_{32}^s H_{28}^s V_{40} \\
& + Q_2 d_2^c N_2^c \Phi_{12} H_{30}^s V_{24}^s H_{35} \\
& + Q_2 d_3^c \Phi_{12} H_{15}^s H_{30}^s V_{24}^s H_{35} \\
& + Q_2 d_3^c \Phi_{12} H_{15}^s H_{30}^s V_7^s V_{25} \\
& + Q_3 d_1^c N_2^c \Phi_{12} H_{30}^s H_{28}^s V_{39} \\
& + Q_3 d_2^c N_1^c \Phi_{12} H_{30}^s H_{28}^s V_{39} \\
& + Q_3 d_2^c \Phi_{12} H_{17}^s H_{30}^s V_5^s V_{27} \\
& + Q_3 d_2^c \Phi_{12} H_{19}^s V_{22}^s V_{34}^s H_{35} \\
& + Q_3 d_2^c \Phi_{12} H_{30}^s H_{39}^s V_4^s V_{33} \\
& + Q_3 d_3^c N_3^c \Phi_{13} H_{16}^s H_{26}^s V_{37} \\
& + Q_3 d_3^c \Phi_{13} H_{15}^s H_{30}^s V_{14}^s V_3 \\
& + Q_3 d_3^c \Phi_{13} H_{19}^s H_{31}^s V_{15}^s V_{37} \\
& + Q_3 d_3^c \Phi_{13} H_{21}^s H_{30}^s V_{19}^s V_{39} \\
& + Q_3 d_3^c \Phi_{13} H_{30}^s V_{31}^s H_{23}^s V_{35} \\
& + Q_1 d_2^c \Phi_{12} H_{29}^s H_{30}^s V_5^s V_{17} \\
& + Q_1 d_2^c N_2^c \Phi_{12} H_{31}^s H_{26}^s V_{37} \\
& + Q_2 d_1^c N_2^c \Phi_{12} H_{30}^s H_{32}^s V_1^s \\
& + Q_2 d_2^c N_1^c \Phi_{12} H_{30}^s H_{32}^s V_1^s \\
& + Q_2 d_2^c N_3^c \Phi_{12} H_{30}^s V_{24}^s H_{35} \\
& + Q_2 d_2^c \Phi_{12} H_{17}^s H_{30}^s V_9^s V_{19} \\
& + Q_2 d_2^c \Phi_{12} H_{19}^s V_{12}^s V_{34}^s H_{35} \\
& + Q_2 d_2^c \Phi_{12} H_{30}^s H_{37}^s V_{34}^s H_{35} \\
& + Q_2 d_3^c N_2^c \Phi_{12} H_{30}^s H_{37}^s H_{35} \\
& + Q_2 d_3^c \Phi_{12} H_{16}^s H_{26}^s V_{37} \\
& + Q_2 d_3^c \Phi_{12} H_{15}^s H_{22}^s H_{30}^s H_{37}^s \\
& + Q_2 d_3^c \Phi_{12} H_{29}^s H_{30}^s V_{12}^s V_{21}^s \\
& + Q_3 d_1^c N_3^c \Phi_{13} H_{30}^s H_{32}^s V_1^s \\
& + Q_3 d_2^c \Phi_{12} H_{17}^s H_{20}^s H_{30}^s H_{37}^s \\
& + Q_3 d_2^c \Phi_{12} H_{19}^s H_{30}^s V_{25}^s V_{35} \\
& + Q_3 d_2^c \Phi_{12} H_{29}^s H_{30}^s V_{11}^s V_{22}^s \\
& + Q_3 d_2^c \Phi_{13} H_{30}^s H_{39}^s V_2^s V_{31}^s \\
& + Q_3 d_3^c N_3^c \Phi_{13} H_{30}^s H_{32}^s V_{21}^s \\
& + Q_3 d_3^c \Phi_{13} H_{17}^s H_{30}^s V_4^s V_{13} \\
& + Q_3 d_3^c \Phi_{13} H_{19}^s H_{32}^s V_{34}^s V_{13} \\
& + Q_3 d_3^c \Phi_{13} H_{30}^s H_{32}^s H_{37}^s H_{39}^s \\
& + Q_3 d_3^c \Phi_{13} H_{37}^s V_{32}^s H_{28}^s V_{40} \\
& + \dots]
\end{aligned}$$

Possible electron mass terms from $\langle h_1 \rangle$:

$$\begin{aligned}
h_1 & [L_3 e_3^c H_{29}^s H_{30}^s \\
& + L_2 e_2^c H_{29}^s H_{30}^s V_{31}^s V_{32}^s \\
& + L_1 e_2^c N_2^c \Phi_{12} H_{30}^s V_4 H_{35} \\
& + L_1 e_3^c N_2^c \Phi_{12} H_{30}^s H_{26} V_{35} \\
& + L_1 e_3^c \Phi_{12} H_{29}^s H_{30}^s V_4 V_{23} \\
& + L_2 e_2^c \Phi_{12} H_{29}^s H_{30}^s V_{14} V_3 \\
& + L_2 e_2^c N_3^c \Phi_{12} H_{30}^s H_{32}^s V_{21} \\
& + L_2 e_2^c \Phi_{12} H_{17}^s H_{30}^s V_4 V_{13} \\
& + L_2 e_2^c \Phi_{12} H_{19}^s H_{32}^s V_{34} V_{13} \\
& + L_2 e_2^c \Phi_{12} H_{30}^s H_{32}^s H_{37}^s H_{39} \\
& + L_2 e_2^c \Phi_{12} H_{37}^s V_{32}^s H_{28} V_{40} \\
& + L_2 e_3^c \Phi_{12} H_{17}^s H_{30}^s V_4 V_{23} \\
& + L_2 e_3^c \Phi_{12} H_{19}^s H_{32}^s V_{34} V_{23} \\
& + L_2 e_3^c \Phi_{12} H_{29}^s H_{30}^s V_{14} V_{23} \\
& + L_3 e_1^c N_2^c \Phi_{12} H_{31}^s H_{28} V_{40} \\
& + L_3 e_1^c \Phi_{12} H_{29}^s H_{30}^s V_7 V_{25} \\
& + L_3 e_2^c N_2^c \Phi_{12} H_{30}^s V_{24} H_{35} \\
& + L_3 e_2^c \Phi_{12} H_{15}^s H_{30}^s V_7 V_{25} \\
& + L_3 e_3^c N_2^c \Phi_{13} H_{30}^s H_{32}^s V_1 \\
& + L_3 e_3^c N_3^c \Phi_{13} H_{30}^s V_{24} H_{35} \\
& + L_3 e_3^c \Phi_{13} H_{17}^s H_{30}^s V_9 V_{19} \\
& + L_3 e_3^c \Phi_{13} H_{19}^s V_{12}^s V_{34} H_{35} \\
& + L_3 e_3^c \Phi_{13} H_{30}^s H_{37}^s H_{42} H_{35} \\
& + L_1 e_2^c \Phi_{12} H_{29}^s H_{30}^s V_4 V_{13} \\
& + L_1 e_3^c N_3^c \Phi_{13} H_{30}^s V_4 H_{35} \\
& + L_1 e_3^c \Phi_{12} H_{29}^s H_{30}^s V_5 V_{27} \\
& + L_2 e_2^c N_3^c \Phi_{12} H_{32}^s H_{32}^s V_1 \\
& + L_2 e_2^c N_3^c \Phi_{12} H_{30}^s V_{24} H_{35} \\
& + L_2 e_2^c \Phi_{12} H_{17}^s H_{30}^s V_9 V_{19} \\
& + L_2 e_2^c \Phi_{12} H_{19}^s V_{12}^s V_{34} H_{35} \\
& + L_2 e_2^c \Phi_{12} H_{30}^s H_{37}^s H_{42} H_{35} \\
& + L_2 e_2^c N_1^c \Phi_{12} H_{30}^s H_{28} V_{39} \\
& + L_2 e_2^c \Phi_{12} H_{17}^s H_{30}^s V_5 V_{27} \\
& + L_2 e_3^c \Phi_{12} H_{19}^s V_{22}^s V_{34} H_{35} \\
& + L_2 e_3^c \Phi_{12} H_{30}^s H_{39}^s V_4 V_{33} \\
& + L_3 e_1^c \Phi_{12} H_{22}^s H_{29}^s H_{30}^s H_{37}^s \\
& + L_3 e_2^c N_2^c \Phi_{12} H_{16}^s H_{26} V_{37} \\
& + L_3 e_2^c \Phi_{12} H_{15}^s H_{22}^s H_{30}^s H_{37}^s \\
& + L_3 e_2^c \Phi_{12} H_{29}^s H_{30}^s V_{12}^s V_{21} \\
& + L_3 e_3^c N_3^c \Phi_{13} H_{16}^s H_{26} V_{37} \\
& + L_3 e_3^c \Phi_{13} H_{15}^s H_{30}^s V_{14} V_3 \\
& + L_3 e_3^c \Phi_{13} H_{19}^s H_{31}^s V_{15} V_{37} \\
& + L_3 e_3^c \Phi_{13} H_{21}^s H_{30}^s V_{19} V_{39} \\
& + L_3 e_3^c \Phi_{13} H_{30}^s V_{31}^s H_{23} V_{35} \\
& + L_1 e_2^c \Phi_{12} H_{29}^s H_{30}^s V_9 V_{19} \\
& + L_1 e_3^c \Phi_{12} H_{20}^s H_{29}^s H_{30}^s H_{37}^s \\
& + L_2 e_2^c N_3^c \Phi_{12} H_{31}^s H_{28} V_{40} \\
& + L_2 e_2^c N_3^c \Phi_{12} H_{16}^s H_{26} V_{37} \\
& + L_2 e_2^c \Phi_{12} H_{15}^s H_{30}^s V_{14} V_3 \\
& + L_2 e_2^c \Phi_{12} H_{19}^s H_{32}^s V_{34} V_{13} \\
& + L_2 e_2^c \Phi_{12} H_{30}^s H_{32}^s H_{37}^s H_{39} \\
& + L_2 e_3^c \Phi_{12} H_{19}^s V_{31}^s H_{23} V_{35} \\
& + L_2 e_3^c \Phi_{12} H_{19}^s H_{31}^s V_{15} V_{37} \\
& + L_2 e_3^c \Phi_{12} H_{21}^s H_{30}^s V_{19} V_{39} \\
& + L_2 e_3^c \Phi_{12} H_{30}^s V_{31}^s H_{23} V_{35} \\
& + L_2 e_3^c \Phi_{12} H_{17}^s H_{20}^s H_{30}^s H_{37}^s \\
& + L_2 e_3^c \Phi_{12} H_{19}^s H_{30}^s V_{25} V_{35} \\
& + L_2 e_3^c \Phi_{12} H_{29}^s H_{30}^s V_{11}^s V_{22} \\
& + L_2 e_3^c \Phi_{13} H_{30}^s H_{39}^s V_2^s V_{31} \\
& + L_3 e_1^c \Phi_{12} H_{29}^s H_{30}^s V_{24} V_3 \\
& + L_3 e_2^c N_2^c \Phi_{12} H_{30}^s H_{32}^s V_{21} \\
& + L_3 e_2^c \Phi_{12} H_{15}^s H_{30}^s V_{24} V_3 \\
& + L_3 e_2^c \Phi_{12} H_{29}^s H_{30}^s V_{24} V_{13} \\
& + L_3 e_3^c N_3^c \Phi_{13} H_{30}^s H_{32}^s V_{21} \\
& + L_3 e_3^c \Phi_{13} H_{17}^s H_{30}^s V_4 V_{13} \\
& + L_3 e_3^c \Phi_{13} H_{19}^s H_{32}^s V_{34} V_{13} \\
& + L_3 e_3^c \Phi_{13} H_{30}^s H_{32}^s H_{37}^s H_{39} \\
& + L_3 e_3^c \Phi_{13} H_{37}^s V_{32}^s H_{28} V_{40} \\
& + \dots]
\end{aligned}$$

Possible down-quark mass terms from $\langle h_2 \rangle$:

$$\begin{aligned}
& h_2 [Q_2 d_2^c \\
& + Q_3 d_3^c V_{34} V_{33} \\
& + Q_1 d_3^c \Phi_{13} V_2^s V_{21}^s \\
& + Q_1 d_1^c H_{29}^s H_{30}^s V_1^s V_2^s \\
& + Q_1 d_2^c N_3^c H_{31}^s H_{26}^s V_{37} \\
& + Q_1 d_2^c H_{19}^s V_2^s V_{34} H_{35} \\
& + Q_1 d_3^c N_2^c H_{31}^s H_{26}^s V_{37} \\
& + Q_1 d_3^c \Phi_{13} \bar{\Phi}_4 V_7 V_{25} \\
& + Q_2 d_3^c N_2^c H_{16}^s H_{26}^s V_{37} \\
& + Q_2 d_3^c H_{15}^s H_{30}^s V_{25} V_{25} \\
& + Q_3 d_1^c \Phi_{13} \Phi_4 V_4 V_{23} \\
& + Q_3 d_2^c N_1^c H_{30}^s H_{28}^s V_{39} \\
& + Q_3 d_2^c H_{17}^s H_{30}^s V_5 V_{27} \\
& + Q_3 d_2^c H_{19}^s V_{22}^s V_{34} H_{35} \\
& + Q_3 d_3^c H_{29}^s H_{30}^s V_{21}^s V_{22} \\
& + Q_3 d_3^c H_{29}^s H_{30}^s V_{25} V_{27} \\
& + Q_1 d_1^c N_2^c \Phi_{13} H_{20}^s H_{31}^s V_{31} \\
& + Q_1 d_1^c \Phi_{13} H_{37}^s V_{32}^s H_{28}^s V_{39} \\
& + Q_1 d_2^c N_2^c \Phi_{13} H_{18}^s H_{20}^s V_{31}^s \\
& + Q_1 d_2^c \Phi_{12} V_{34} V_{33} V_5 V_{17} \\
& + Q_1 d_2^c \Phi_{12} H_{23}^s H_{26}^s V_{27} V_{35} \\
& + Q_1 d_2^c \Phi_{13} H_{20}^s H_{21}^s V_7 V_{15} \\
& + Q_1 d_2^c \Phi_{13} V_{11}^s V_{32}^s V_4 V_{33} \\
& + Q_1 d_2^c \Phi_{13} V_9 V_{40} V_{17} V_{35} \\
& + Q_1 d_2^c \bar{\Phi}_4^c H_{17}^s H_{30}^s V_1^s V_2^s \\
& + Q_1 d_2^c \bar{\Phi}_4^c H_{19}^s H_{32}^s V_{34} V_3 \\
& + Q_1 d_3^c \Phi_{12} V_2^s V_{21}^s V_{34} V_{33} \\
& + Q_1 d_3^c \Phi_{13} H_{15}^s H_{31}^s V_{11}^s V_{22}^s \\
& + Q_1 d_3^c \Phi_{13} H_{16}^s H_{42}^s V_4 V_{34} \\
& + Q_1 d_3^c \Phi_{13} H_{31}^s H_{38}^s V_{15} V_{35} \\
& + Q_1 d_3^c \Phi_{13} V_4 H_{35}^s H_{23}^s V_{27} \\
& + Q_1 d_3^c \bar{\Phi}_4^c H_{29}^s H_{30}^s V_7 V_{25} \\
& + Q_2 d_1^c N_3^c \bar{\Phi}_{56}^c H_{30}^s H_{28}^s V_{39} \\
& + Q_2 d_1^c \Phi_{12} V_{12}^s V_{12}^s V_{34} V_{33} \\
& + Q_2 d_1^c \Phi_{13} V_{12}^s V_{31}^s V_{34} V_3 \\
& + Q_2 d_1^c \Phi_4^c H_{15}^s H_{30}^s V_5 V_7 \\
& + Q_2 d_3^c \Phi_{12} V_{12}^s V_{21}^s V_{34} V_{33} \\
& + Q_2 d_3^c \Phi_{13} H_{15}^s H_{16}^s V_{14} V_{23} \\
& + Q_2 d_3^c \Phi_4^c H_{15}^s H_{30}^s V_2^s V_{21}^s \\
& + Q_2 d_3^c \bar{\Phi}_{56}^c H_{16}^s H_{21}^s H_{29}^s H_{36}^s \\
& + Q_1 d_3^c \Phi_{13} V_9 V_{29} V_{29} \\
& + Q_1 d_3^c H_{29}^s H_{30}^s V_4 V_3 \\
& + Q_1 d_3^c H_{15}^s H_{19}^s H_{20}^s H_{31}^s \\
& + Q_1 d_3^c \bar{\Phi}_4^c H_{17}^s H_{30}^s V_5 V_7 \\
& + Q_1 d_3^c N_3^c \Phi_{13} H_{16}^s H_{26}^s V_{35} \\
& + Q_1 d_2^c N_3^c \Phi_{13} H_{20}^s H_{31}^s V_{31}^s \\
& + Q_1 d_2^c N_3^c \bar{\Phi}_4^c H_{31}^s H_{28}^s V_{40} \\
& + Q_1 d_2^c \Phi_{12} H_{25}^s V_{30} H_{26}^s V_{35} \\
& + Q_1 d_2^c \Phi_{13} H_{18}^s H_{19}^s V_{15} V_{35} \\
& + Q_1 d_2^c \Phi_{13} H_{20}^s V_{31}^s H_{25}^s V_9 \\
& + Q_1 d_2^c \Phi_{13} H_{42}^s V_{13} H_{28}^s V_9 \\
& + Q_1 d_2^c \bar{\Phi}_4^c H_{21}^s H_{30}^s V_5 V_{35} \\
& + Q_1 d_2^c \bar{\Phi}_4^c H_{17}^s H_{30}^s V_4 V_3 \\
& + Q_1 d_3^c N_3^c \Phi_{13} H_{16}^s H_{26}^s V_{35} \\
& + Q_1 d_3^c \Phi_{12} V_{34} V_{33} V_9 V_{29} \\
& + Q_1 d_3^c \Phi_{13} H_{15}^s H_{31}^s V_{14} V_{23} \\
& + Q_1 d_3^c \Phi_{13} H_{18}^s H_{29}^s V_{19} V_{30} \\
& + Q_1 d_3^c \Phi_{13} V_{21}^s V_{32}^s V_4 V_{33} \\
& + Q_1 d_3^c \bar{\Phi}_4^c H_{22}^s H_{29}^s H_{30}^s H_{37}^s \\
& + Q_1 d_3^c \bar{\Phi}_{56}^c H_{21}^s H_{29}^s H_{31}^s H_{36}^s \\
& + Q_2 d_1^c \Phi_{12} H_{30}^s H_{37}^s V_{29} V_{39} \\
& + Q_2 d_1^c \Phi_{12} V_{34} V_{33} V_7 V_{15} \\
& + Q_2 d_1^c \bar{\Phi}_4^c H_{15}^s H_{30}^s V_1^s V_2^s \\
& + Q_2 d_3^c N_3^c \bar{\Phi}_{56}^c H_{30}^s H_{28}^s V_{39} \\
& + Q_2 d_3^c \Phi_{12} V_{24}^s V_{34}^s V_{13} V_{33} \\
& + Q_2 d_3^c \Phi_{13} H_{16}^s H_{38}^s V_{15} V_{35} \\
& + Q_2 d_3^c \Phi_4^c H_{15}^s H_{30}^s V_9 V_{29} \\
& + Q_2 d_3^c \bar{\Phi}_{56}^c H_{21}^s H_{31}^s V_{29} V_{40} \\
& + Q_3 d_1^c \Phi_{13} V_1^s V_{22}^s \\
& + Q_1 d_1^c H_{29}^s H_{30}^s V_5 V_7 \\
& + Q_1 d_2^c H_{19}^s H_{31}^s V_5 V_{37} \\
& + Q_1 d_2^c H_{29}^s V_{32}^s H_{26}^s V_{27} \\
& + Q_1 d_3^c \bar{\Phi}_4^c V_{24}^s V_3 \\
& + Q_1 d_3^c H_{29}^s H_{30}^s V_9 V_{29} \\
& + Q_2 d_3^c H_{15}^s H_{30}^s V_{24} V_3 \\
& + Q_3 d_1^c \Phi_{13} \bar{\Phi}_4^c H_{20}^s H_{37}^s \\
& + Q_3 d_1^c H_{29}^s H_{30}^s V_1^s V_{22}^s \\
& + Q_3 d_2^c H_{17}^s H_{30}^s V_4 V_{23} \\
& + Q_3 d_2^c H_{19}^s H_{32}^s V_{34} V_{23} \\
& + Q_3 d_3^c H_{29}^s H_{30}^s H_{36}^s H_{37}^s \\
& + Q_3 d_3^c H_{29}^s H_{30}^s V_{29} V_{30}
\end{aligned}$$

Possible down–quark mass terms from $\langle h_2 \rangle$ continued:

$$\begin{aligned}
& +Q_2 d_3^c \overline{\Phi}'_{56} H_{17}^s H_{30}^s V_4 V_{23} & +Q_2 d_3^c \overline{\Phi}'_{56} H_{17}^s H_{30}^s V_5 V_{27} & +Q_2 d_3^c \overline{\Phi}'_{56} H_{19}^s H_{30}^s V_{25} V_{35} \\
& +Q_2 d_3^c \overline{\Phi}'_{56} H_{19}^s H_{32}^s V_{34} V_{23} & +Q_2 d_3^c \overline{\Phi}'_{56} H_{19}^s V_{22}^s V_{34} H_{35} & +Q_2 d_3^c \overline{\Phi}'_{56} H_{30}^s H_{39}^s V_4 V_{33} \\
& +Q_3 d_1^c N_2^c \overline{\Phi}'_4 H_{30}^s H_{26} V_{35} & +Q_3 d_1^c N_3^c \overline{\Phi}'_{23} H_{30}^s H_{32}^s V_1^s & +Q_3 d_1^c \Phi_{12} V_1^s V_{22}^s V_{34} V_{33} \\
& +Q_3 d_1^c \Phi_{13} V_{22}^s V_{31}^s V_{34} V_3 & +Q_3 d_1^c \Phi_{13} V_{30} V_{39} V_7 V_{37} & +Q_3 d_1^c \Phi_{13} V_{30} V_{40} V_7 V_{35} \\
& +Q_3 d_1^c \Phi_4 H_{20}^s H_{29}^s H_{30}^s H_{37}^s & +Q_3 d_1^c \Phi_4 H_{29}^s H_{30}^s V_4 V_{23} & +Q_3 d_1^c \Phi_4 H_{29}^s H_{30}^s V_5 V_{27} \\
& +Q_3 d_1^c \overline{\Phi}'_{56} H_{19}^s H_{29}^s H_{31}^s H_{36}^s & +Q_3 d_2^c N_1^c \overline{\Phi}'_4 H_{30}^s H_{26} V_{35} & +Q_3 d_2^c N_2^c \overline{\Phi}'_{56} H_{16}^s H_{26} V_{37} \\
& +Q_3 d_2^c \Phi_{12} V_{11}^s V_{22}^s V_{34} V_{33} & +Q_3 d_2^c \Phi_{12} V_{14} V_{34} V_{23} V_{33} & +Q_3 d_2^c \Phi_{13} H_{19}^s H_{36}^s V_9 V_{19} \\
& +Q_3 d_2^c \Phi_{13} H_{19}^s H_{20}^s V_{24} V_{13} & +Q_3 d_2^c \Phi_{13} H_{19}^s H_{36}^s V_4 V_{13} & +Q_3 d_2^c \Phi_{13} H_{20}^s V_{31}^s H_{25} V_{30} \\
& +Q_3 d_2^c \Phi_{13} H_{20}^s H_{38}^s V_1^s V_{12} & +Q_3 d_2^c \Phi_{13} H_{20}^s H_{38}^s V_7 V_{15} & +Q_3 d_2^c \Phi_{13} H_{39}^s V_{12}^s H_{26} V_{27} \\
& +Q_3 d_2^c \Phi_{13} H_{20}^s V_{31}^s H_{23} V_{27} & +Q_3 d_2^c \Phi_{13} H_{39}^s V_{12}^s H_{28} V_{30} & +Q_3 d_2^c \Phi_{13} V_{20} V_{39} V_{27} V_{37} \\
& +Q_3 d_2^c \Phi_{13} H_{42} V_{13} H_{28} V_{30} & +Q_3 d_2^c \Phi_{13} H_{42} V_{13} H_{26} V_{27} & +Q_3 d_2^c \Phi_{13} V_{30} V_{40} V_{17} V_{35} \\
& +Q_3 d_2^c \Phi_{13} V_{20} V_{40} V_{27} V_{35} & +Q_3 d_2^c \Phi_{13} V_{30} V_{39} V_{17} V_{37} & +Q_3 d_2^c \overline{\Phi}'_4 H_{30}^s H_{38}^s V_5 V_{35} \\
& +Q_3 d_2^c \overline{\Phi}'_{23} H_{30}^s H_{39}^s V_2^s V_{31}^s & +Q_3 d_2^c \overline{\Phi}'_{23} H_{19}^s H_{35}^s V_{23} V_{33} & +Q_3 d_2^c \overline{\Phi}'_4 H_{19}^s H_{30}^s V_{29} V_{39} \\
& +Q_3 d_2^c \overline{\Phi}'_4 H_{30}^s H_{42} V_4 V_{34} & +Q_3 d_2^c \overline{\Phi}'_4 H_{17}^s H_{30}^s V_1^s V_{22}^s & +Q_3 d_2^c \overline{\Phi}'_{56} H_{18}^s H_{19}^s H_{29}^s H_{36}^s \\
& +Q_3 d_2^c \overline{\Phi}'_4 H_{30}^s V_1^s H_{42} V_{33} & +Q_3 d_2^c \overline{\Phi}'_{56} H_{17}^s H_{20}^s H_{31}^s H_{38}^s & +Q_3 d_2^c \overline{\Phi}'_{56} H_{15}^s H_{30}^s V_7 V_{25} \\
& +Q_3 d_2^c \overline{\Phi}'_{56} H_{15}^s H_{22}^s H_{30}^s H_{37}^s & +Q_3 d_2^c \overline{\Phi}'_{56} H_{15}^s H_{30}^s V_{24} V_3 & +Q_3 d_2^c \overline{\Phi}'_{23} H_{30}^s H_{32}^s V_{21}^s \\
& +Q_3 d_3^c N_1^c \overline{\Phi}'_{23} H_{30}^s H_{32}^s V_1^s & +Q_3 d_3^c N_3^c \overline{\Phi}'_{23} H_{16}^s H_{26} V_{37} & +Q_3 d_3^c \overline{\Phi}'_{23} H_{17}^s H_{30}^s V_4 V_{13} \\
& +Q_3 d_3^c N_3^c \overline{\Phi}'_{23} H_{30}^s V_{24} H_{35} & +Q_3 d_3^c \overline{\Phi}'_{23} H_{15}^s H_{30}^s V_{14} V_3 & +Q_3 d_3^c \overline{\Phi}'_{23} H_{19}^s H_{32}^s V_{34} V_{13} \\
& +Q_3 d_3^c \overline{\Phi}'_{23} H_{17}^s H_{30}^s V_9 V_{19} & +Q_3 d_3^c \overline{\Phi}'_{23} H_{19}^s H_{31}^s V_{15} V_{37} & +Q_3 d_3^c \overline{\Phi}'_{23} H_{30}^s H_{32}^s H_{37}^s H_{39}^s \\
& +Q_3 d_3^c \overline{\Phi}'_{23} H_{19}^s V_{12}^s V_{34} H_{35} & +Q_3 d_3^c \overline{\Phi}'_{23} H_{21}^s H_{30}^s V_{19} V_{39} & +Q_3 d_3^c \overline{\Phi}'_{23} H_{37}^s V_{32}^s H_{28} V_{40} \\
& +Q_3 d_3^c \overline{\Phi}'_{23} H_{30}^s H_{37}^s H_{42} H_{35} & +Q_3 d_3^c \overline{\Phi}'_{23} H_{30}^s V_{31}^s H_{23} V_{35} & +Q_3 d_3^c \overline{\Phi}'_{56} H_{25}^s H_{26}^s H_{26}^s \\
& +Q_3 d_3^c \overline{\Phi}'_{56} H_{29}^s H_{31}^s H_{36}^s H_{38}^s & +Q_3 d_3^c \overline{\Phi}'_{56} H_{25}^s H_{25}^s H_{26}^s H_{26}^s & +Q_3 d_3^c \overline{\Phi}'_{56} H_{25}^s H_{28}^s H_{23}^s H_{26}^s \\
& +Q_3 d_3^c \overline{\Phi}'_{56} H_{28} H_{28} H_{23} H_{23} & +\dots
\end{aligned}$$

Possible electron mass terms from $\langle h_2 \rangle$:

$$\begin{aligned}
& h_2 [L_2 e_2^c \\
& + L_3 e_3^c V_{34} V_{33} \\
& + L_1 e_3^c \Phi_{13} H_{20}^s H_{37}^s \\
& + L_3 e_1^c \Phi_{13} H_{22}^s H_{37}^s \\
& + L_1 e_1^c H_{29}^s H_{30}^s V_2^s V_2^s \\
& + L_1 e_2^c H_{15}^s H_{30}^s V_1^s V_2^s \\
& + L_1 e_3^c N_2^c H_{30}^s H_{26}^s V_{35} \\
& + L_1 e_3^c H_{29}^s H_{30}^s V_4 V_{23} \\
& + L_2 e_1^c H_{16}^s H_{19}^s H_{20}^s H_{29}^s \\
& + L_2 e_1^c H_{17}^s H_{30}^s V_5 V_7 \\
& + L_2 e_3^c H_{17}^s H_{20}^s H_{30}^s H_{37}^s \\
& + L_2 e_3^c H_{19}^s H_{30}^s V_{25} V_{35} \\
& + L_2 e_3^c H_{30}^s H_{39}^s V_4 V_{33} \\
& + L_3 e_1^c \Phi_{13} \Phi_4 V_9 V_{29} \\
& + L_3 e_1^c H_{29}^s H_{30}^s V_7 V_{25} \\
& + L_3 e_2^c H_{15}^s H_{30}^s V_{24} V_3 \\
& + L_3 e_3^c H_{29}^s H_{30}^s V_{21} V_{22} \\
& + L_3 e_3^c H_{29}^s H_{30}^s V_{25} V_{27} \\
& + L_1 e_1^c N_2^c \Phi_{13} H_{20}^s H_{31}^s V_{31}^s \\
& + L_1 e_1^c \Phi_{13} H_{37}^s V_{32}^s H_{28}^s V_{39} \\
& + L_1 e_2^c \Phi_{12} H_{32}^s H_{37}^s V_{34} V_{23} \\
& + L_1 e_2^c \Phi_{12} V_{34} V_{33} V_9 V_{19} \\
& + L_1 e_3^c N_3^c \overline{\Phi}_{23} H_{30}^s V_4 H_{35} \\
& + L_1 e_3^c \Phi_{12} V_{34} V_{33} V_5 V_{27} \\
& + L_1 e_3^c \Phi_{13} V_9 V_{40} V_{27} V_{35} \\
& + L_2 e_1^c N_2^c \Phi_{12} H_{22}^s H_{28}^s V_{30} \\
& + L_2 e_1^c N_3^c \Phi_{13} H_{31}^s H_{36}^s V_{31}^s \\
& + L_2 e_1^c \Phi_{12} H_{25}^s H_{28}^s V_{30} V_{39} \\
& + L_2 e_1^c \Phi_{13} H_{18}^s H_{19}^s V_{19} V_{39} \\
& + L_2 e_1^c \Phi_{13} H_{20}^s V_{31}^s H_{23} V_7 \\
& + L_2 e_1^c \Phi_{13} H_{39}^s V_{12}^s H_{26}^s V_7 \\
& + L_2 e_1^c \Phi_{13} V_{20} V_{39} V_7 V_{37} \\
& + L_2 e_1^c \Phi_4 H_{29}^s V_{32}^s H_{26}^s V_{27} \\
& + L_2 e_1^c \Phi_4' H_{19}^s V_2^s V_{34} H_{35} \\
& + L_2 e_3^c N_2^c \Phi_{56}^s H_{16}^s H_{26}^s V_{35} \\
& + L_2 e_2^c \Phi_{12} H_{37}^s V_{22}^s V_{34} H_{35} \\
& + L_1 e_2^c \Phi_{13} V_2^s V_{31}^s V_{34} V_{13} \\
& + L_1 e_3^c \Phi_{12} H_{20}^s H_{37}^s V_{34} V_{33} \\
& + L_1 e_3^c \Phi_{13} V_2^s V_{31}^s V_{34} V_{23} \\
& + L_1 e_3^c \overline{\Phi}_4 H_{29}^s H_{30}^s V_1^s V_{22} \\
& + L_2 e_1^c N_2^c \Phi_{12} H_{22}^s H_{26}^s V_{27} \\
& + L_2 e_1^c N_3^c \Phi_4' H_{31}^s H_{26}^s V_{37} \\
& + L_2 e_1^c \Phi_{12} H_{25}^s V_{39} H_{26}^s V_{27} \\
& + L_2 e_1^c \Phi_{13} H_{19}^s H_{22}^s V_4 V_{13} \\
& + L_2 e_1^c \Phi_{13} H_{31}^s H_{39}^s V_{22}^s V_{31}^s \\
& + L_2 e_1^c \Phi_{13} V_1^s V_{32}^s V_{14} V_{33} \\
& + L_2 e_1^c \Phi_{13} V_{20} V_{40} V_7 V_{35} \\
& + L_2 e_1^c \Phi_4' H_{15}^s H_{19}^s H_{20}^s H_{31}^s \\
& + L_2 e_1^c \overline{\Phi}_4 H_{19}^s H_{35}^s V_3 V_{33} \\
& + L_2 e_3^c \Phi_{12} V_{11}^s V_{22}^s V_{34} V_{33} \\
& + L_2 e_3^c \Phi_{13} H_{19}^s H_{20}^s V_{24} V_{13} \\
& + L_2 e_3^c \Phi_{13} H_{20}^s H_{38}^s V_1^s V_{12}^s \\
& + L_2 e_3^c \Phi_{13} H_{20}^s V_{31}^s H_{23} V_{27} \\
& + L_2 e_3^c \Phi_{13} H_{42}^s V_{13} H_{28}^s V_{30} \\
& + L_2 e_3^c \Phi_{13} V_{20} V_{40} V_{27} V_{35} \\
& + L_2 e_3^c \overline{\Phi}_{23} H_{30}^s H_{39}^s V_2^s V_{31}^s \\
& + L_1 e_3^c \Phi_{13} V_4 V_{23} \\
& + L_3 e_1^c \Phi_{13} V_{24} V_3 \\
& + L_1 e_1^c H_{29}^s H_{30}^s V_4 V_{27} \\
& + L_1 e_2^c H_{15}^s H_{30}^s V_5 V_7 \\
& + L_1 e_3^c H_{20}^s H_{29}^s H_{30}^s H_{37}^s \\
& + L_2 e_1^c N_3^c H_{31}^s H_{28} V_{40} \\
& + L_2 e_1^c H_{17}^s H_{30}^s V_4 V_3 \\
& + L_2 e_3^c N_1^c H_{30}^s H_{28} V_{39} \\
& + L_2 e_3^c H_{17}^s H_{30}^s V_5 V_{27} \\
& + L_2 e_3^c H_{19}^s V_{22} V_{34} H_{35} \\
& + L_3 e_1^c \Phi_4 V_2^s V_{21} \\
& + L_3 e_1^c H_{29}^s H_{30}^s V_{24} V_3 \\
& + L_3 e_2^c H_{15}^s H_{22}^s H_{30}^s H_{37}^s \\
& + L_3 e_3^c H_{29}^s H_{30}^s H_{36}^s H_{37}^s \\
& + L_3 e_3^c H_{29}^s H_{30}^s V_{29} V_{30} \\
& + L_1 e_1^c \Phi_{13} H_{19}^s H_{31}^s V_{15} V_{35} \\
& + L_1 e_2^c \Phi_{12} H_{30}^s H_{37}^s V_{25} V_{35} \\
& + L_1 e_2^c \Phi_{12} V_4 V_{34} V_{13} V_{33} \\
& + L_1 e_3^c N_2^c \overline{\Phi}_4 H_{30}^s H_{28} V_{39} \\
& + L_1 e_3^c \Phi_{12} V_4 V_{34} V_{23} V_{33} \\
& + L_1 e_3^c \Phi_{13} V_9 V_{39} V_{27} V_{37} \\
& + L_1 e_3^c \overline{\Phi}_{56} H_{20}^s H_{29}^s H_{31}^s H_{38}^s \\
& + L_2 e_1^c N_2^c \Phi_{12} H_{36}^s H_{26}^s V_7 \\
& + L_2 e_1^c \Phi_{12} V_{14} V_{34} V_3 V_{33} \\
& + L_2 e_1^c \Phi_{12} H_{28}^s V_{39} H_{23}^s V_{27} \\
& + L_2 e_1^c \Phi_{13} H_{19}^s H_{22}^s V_9 V_{19} \\
& + L_2 e_1^c \Phi_{13} H_{31}^s V_{31}^s H_{42}^s V_{23} \\
& + L_2 e_1^c \Phi_{13} H_{42}^s V_{13} H_{26}^s V_7 \\
& + L_2 e_1^c \Phi_4 H_{29}^s V_{32}^s H_{28}^s V_{30} \\
& + L_2 e_1^c \Phi_4' H_{19}^s H_{31}^s V_5 V_{37} \\
& + L_2 e_3^c N_1^c \Phi_4 H_{30}^s H_{26}^s V_{35} \\
& + L_2 e_3^c \Phi_{12} V_{14} V_{34} V_{23} V_{33} \\
& + L_2 e_3^c \Phi_{13} H_{19}^s H_{36}^s V_4 V_{13} \\
& + L_2 e_3^c \Phi_{13} H_{38}^s H_{38}^s V_7 V_{15} \\
& + L_2 e_3^c \Phi_{13} H_{39}^s V_{12}^s H_{28}^s V_{30} \\
& + L_2 e_3^c \Phi_{13} H_{42}^s V_{13} H_{26}^s V_{27} \\
& + L_2 e_3^c \Phi_{13} V_{30} V_{39} V_{17} V_{37} \\
& + L_2 e_3^c \Phi_4 H_{30}^s H_{38}^s V_5 V_{35}
\end{aligned}$$

Possible electron mass terms from $\langle h_2 \rangle$ continued:

$$\begin{aligned}
& +L_2 e_3^c \Phi_4 H_{30}^s H_{42} V_4 V_{34} & +L_2 e_3^c \bar{\Phi}_4 H_{17}^s H_{30}^s V_1^s V_{22} & +L_2 e_3^c \bar{\Phi}_4 H_{19}^s H_{30}^s V_{29} V_{39} \\
& +L_2 e_3^c \bar{\Phi}_4 H_{30}^s V_1^s H_{42} V_{33} & +L_2 e_3^c \bar{\Phi}_4' H_{19}^s H_{35} V_{23} V_{33} & +L_2 e_3^c \bar{\Phi}_{56} H_{17}^s H_{20}^s H_{31}^s H_{38}^s \\
& +L_2 e_3^c \bar{\Phi}_{56} H_{18}^s H_{19}^s H_{29}^s H_{36}^s & +L_2 e_3^c \bar{\Phi}_{56}' H_{15}^s H_{22}^s H_{30}^s H_{37}^s & +L_2 e_3^c \bar{\Phi}_{56}' H_{15}^s H_{30}^s V_{24} V_3 \\
& +L_2 e_3^c \bar{\Phi}_{56}' H_{15}^s H_{30}^s V_7 V_{25} & +L_3 e_1^c N_1^c \Phi_{13} H_{16}^s H_{28} V_{39} & +L_3 e_1^c N_2^c \Phi_{13} H_{31}^s H_{36}^s V_{31}^s \\
& +L_3 e_1^c N_2^c \Phi_4 H_{31}^s H_{26} V_{37} & +L_3 e_1^c \Phi_{12} H_{22}^s H_{37}^s V_{34} V_{33} & +L_3 e_1^c \Phi_{12} V_{24} V_{34} V_3 V_{33} \\
& +L_3 e_1^c \Phi_{12} V_{34} V_{33} V_7 V_{25} & +L_3 e_1^c \Phi_{13} H_{16}^s H_{17}^s H_{20}^s H_{37}^s & +L_3 e_1^c \Phi_{13} H_{16}^s H_{17}^s V_4 V_{23} \\
& +L_3 e_1^c \Phi_{13} H_{16}^s H_{17}^s V_5 V_{27} & +L_3 e_1^c \Phi_{13} H_{16}^s H_{19}^s V_{25} V_{35} & +L_3 e_1^c \Phi_{13} H_{16}^s H_{29}^s V_{11}^s V_{22} \\
& +L_3 e_1^c \Phi_{13} H_{16}^s H_{29}^s V_{14} V_{23} & +L_3 e_1^c \Phi_{13} H_{16}^s H_{39}^s V_4 V_{33} & +L_3 e_1^c \Phi_{13} H_{17}^s H_{31}^s V_{19} V_{30} \\
& +L_3 e_1^c \Phi_{13} H_{17}^s H_{31}^s V_{15} V_{27} & +L_3 e_1^c \Phi_{13} H_{31}^s H_{38}^s V_{19} V_{39} & +L_3 e_1^c \Phi_{13} H_{32}^s V_1^s H_{25} V_{30} \\
& +L_3 e_1^c \Phi_{13} H_{32}^s V_1^s H_{23} V_{27} & +L_3 e_1^c \Phi_{13} V_1^s V_{32}^s V_{24} V_{33} & +L_3 e_1^c \Phi_4 H_{29}^s H_{30}^s V_2^s V_{21} \\
& +L_3 e_1^c \Phi_4 H_{29}^s H_{30}^s V_9 V_{29} & +L_3 e_1^c \bar{\Phi}_{56} H_{22}^s H_{29}^s H_{31}^s H_{38}^s & +L_3 e_2^c N_1^c \bar{\Phi}_{56}^s H_{30}^s H_{28} V_{39} \\
& +L_3 e_2^c N_2^c \bar{\Phi}_4^s H_{16}^s H_{28} V_{40} & +L_3 e_2^c \Phi_{12} V_{12}^s V_{21}^s V_{34} V_{33} & +L_3 e_2^c \Phi_{12} V_{24} V_{34} V_{13} V_{33} \\
& +L_3 e_2^c \Phi_{13} H_{15}^s H_{16}^s V_{11}^s V_{22} & +L_3 e_2^c \Phi_{13} H_{15}^s H_{16}^s V_{14} V_{23} & +L_3 e_2^c \Phi_{13} H_{16}^s H_{38}^s V_{15} V_{35} \\
& +L_3 e_2^c \Phi_{13} H_{18}^s H_{37}^s V_{19} V_{40} & +L_3 e_2^c \Phi_4 H_{15}^s H_{30}^s V_2^s V_{21}^s & +L_3 e_2^c \Phi_4 H_{15}^s H_{30}^s V_9 V_{29} \\
& +L_3 e_2^c \bar{\Phi}_{56} H_{15}^s H_{22}^s H_{31}^s H_{38}^s & +L_3 e_2^c \bar{\Phi}_{56} H_{16}^s H_{21}^s H_{29}^s H_{36}^s & +L_3 e_2^c \bar{\Phi}_{56} H_{21}^s H_{31}^s V_{29} V_{40} \\
& +L_3 e_2^c \bar{\Phi}_{56}' H_{17}^s H_{20}^s H_{30}^s H_{37}^s & +L_3 e_2^c \bar{\Phi}_{56}' H_{17}^s H_{30}^s V_4 V_{23} & +L_3 e_2^c \bar{\Phi}_{56}' H_{17}^s H_{30}^s V_5 V_{27} \\
& +L_3 e_2^c \bar{\Phi}_{56}' H_{19}^s H_{30}^s V_{25} V_{35} & +L_3 e_2^c \bar{\Phi}_{56}' H_{19}^s H_{32}^s V_{34} V_{23} & +L_3 e_2^c \bar{\Phi}_{56}' H_{19}^s V_{22}^s V_{34} H_{35} \\
& +L_3 e_2^c \bar{\Phi}_{56}' H_{30}^s H_{39}^s V_4 V_{33} & +L_3 e_3^c N_1^c \bar{\Phi}_{23} H_{30}^s H_{32}^s V_1^s & +L_3 e_3^c N_3^c \bar{\Phi}_{23} H_{16}^s H_{26} V_{37} \\
& +L_3 e_3^c N_3^c \bar{\Phi}_{23} H_{30}^s H_{32}^s V_{21}^s & +L_3 e_3^c N_3^c \bar{\Phi}_{23} H_{30}^s V_{24} H_{35} & +L_3 e_3^c \bar{\Phi}_{23} H_{15}^s H_{30}^s V_{14} V_3 \\
& +L_3 e_3^c \bar{\Phi}_{23} H_{17}^s H_{30}^s V_4 V_{13} & +L_3 e_3^c \bar{\Phi}_{23} H_{17}^s H_{30}^s V_9 V_{19} & +L_3 e_3^c \bar{\Phi}_{23} H_{19}^s H_{31}^s V_{15} V_{37} \\
& +L_3 e_3^c \bar{\Phi}_{23} H_{19}^s H_{32}^s V_{34} V_{13} & +L_3 e_3^c \bar{\Phi}_{23} H_{19}^s V_{12}^s V_{34} H_{35} & +L_3 e_3^c \bar{\Phi}_{23} H_{21}^s H_{30}^s V_{19} V_{39} \\
& +L_3 e_3^c \bar{\Phi}_{23} H_{30}^s H_{32}^s H_{37}^s H_{39}^s & +L_3 e_3^c \bar{\Phi}_{23} H_{30}^s H_{37}^s H_{42} H_{35} & +L_3 e_3^c \bar{\Phi}_{23} H_{30}^s V_{31}^s H_{23} V_{35} \\
& +L_3 e_3^c \bar{\Phi}_{23} H_{37}^s V_{32}^s H_{28} V_{40} & +L_3 e_3^c \bar{\Phi}_{56} H_{29}^s H_{31}^s H_{36}^s H_{38}^s & +L_3 e_3^c \bar{\Phi}_{56} H_{25} H_{25} H_{26} H_{26} \\
& +L_3 e_3^c \bar{\Phi}_{56} H_{25} H_{28} H_{23} H_{26} & +L_3 e_3^c \bar{\Phi}_{56} H_{28} H_{28} H_{23} H_{23} & + \dots
\end{aligned}$$

Possible down-quark mass terms from $\langle h_3 \rangle$:

$$\begin{aligned}
& h_3 [Q_3 d_3^c \\
& + Q_2 d_2^c V_{31}^s V_{32}^s \\
& + Q_1 d_2^c \Phi_{12} V_2^s V_{11}^s \\
& + Q_2 d_1^c \Phi_{12} V_7 V_{15} \\
& + Q_1 d_2^c \Phi_{12} \bar{\Phi}'_4 V_{14} V_3 \\
& + Q_2 d_1^c \Phi_{12} \bar{\Phi}'_4 V_4 V_{13} \\
& + Q_3 d_2^c N_3^c H_{30}^s H_{32}^s V_{11}^s \\
& + Q_1 d_1^c N_2^c \Phi_{12} H_{20}^s H_{31}^s V_{31}^s \\
& + Q_1 d_1^c \Phi_{12} H_{37}^s V_{32}^s H_{28} V_{39} \\
& + Q_1 d_2^c N_3^c \Phi_{23} H_{31}^s H_{26} V_{37} \\
& + Q_1 d_2^c \Phi_{12} H_{20}^s H_{21}^s V_7 V_{15} \\
& + Q_1 d_2^c \Phi_{12} V_{11}^s V_{32}^s V_4 V_{33} \\
& + Q_1 d_2^c \Phi_{12} V_9 V_{40} V_{17} V_{35} \\
& + Q_1 d_2^c \Phi_{23} H_{29}^s H_{30}^s V_2^s V_{11}^s \\
& + Q_1 d_2^c \Phi_{23} H_{29}^s V_{32}^s H_{26} V_{27} \\
& + Q_1 d_3^c N_2^c \Phi_{23} H_{31}^s H_{26} V_{37} \\
& + Q_1 d_3^c \Phi_{12} H_{15}^s H_{31}^s V_{14} V_{23} \\
& + Q_1 d_3^c \Phi_{12} H_{18}^s H_{29}^s V_{19} V_{30} \\
& + Q_1 d_3^c \Phi_{12} V_{21}^s V_{32}^s V_4 V_{33} \\
& + Q_1 d_3^c \Phi_{13} H_{32}^s H_{37}^s V_{11}^s V_{32}^s \\
& + Q_1 d_3^c \Phi_{13} V_{31}^s V_{32}^s V_9 V_{29} \\
& + Q_1 d_3^c \Phi_4 H_{15}^s V_{32}^s H_{28} V_9 \\
& + Q_2 d_1^c N_2^c \Phi_{12} H_{16}^s H_{20}^s V_{31}^s \\
& + Q_2 d_1^c \Phi_{12} V_{12}^s V_{31}^s V_{34} V_3 \\
& + Q_2 d_1^c \Phi_{23} H_{29}^s H_{30}^s V_2^s V_{12}^s \\
& + Q_2 d_2^c N_1^c \Phi_{23} H_{30}^s H_{32}^s V_1^s \\
& + Q_2 d_2^c N_3^c \Phi_{23} H_{30}^s V_{24} H_{35} \\
& + Q_2 d_2^c \Phi_{23} H_{17}^s H_{30}^s V_9 V_{19} \\
& + Q_2 d_2^c \Phi_{23} H_{19}^s V_{12}^s V_{34} H_{35} \\
& + Q_2 d_2^c \Phi_{23} H_{30}^s H_{37}^s H_{42} H_{35} \\
& + Q_2 d_3^c N_2^c \Phi_{23} H_{16}^s H_{26} V_{37} \\
& + Q_2 d_3^c N_3^c \bar{\Phi}'_5 H_{30}^s H_{32}^s V_{11}^s \\
& + Q_2 d_3^c \Phi_{12} H_{15}^s H_{16}^s V_{14} V_{23} \\
& + Q_2 d_3^c \Phi_{13} V_{12}^s V_{21}^s V_{31}^s V_{32}^s \\
& + Q_2 d_3^c \Phi_{23} H_{15}^s H_{30}^s V_{24} V_3 \\
& + Q_2 d_3^c \Phi_{23} H_{29}^s H_{30}^s V_{24} V_{13} \\
& + Q_2 d_3^c \bar{\Phi}'_5 H_{30}^s H_{39}^s V_2^s V_{31}^s \\
& + Q_3 d_1^c N_3^c \bar{\Phi}'_4 H_{30}^s V_4 H_{35} \\
& + Q_3 d_1^c \Phi_{12} V_{30}^s V_{40} V_7 V_{35} \\
& + Q_3 d_1^c \bar{\Phi}'_5 H_{19}^s H_{31}^s H_{32}^s H_{39}^s
\end{aligned}
\begin{aligned}
& + Q_1 d_2^c \Phi_{12} V_5 V_{17} \\
& + Q_2 d_1^c \Phi_{12} V_1^s V_{12}^s \\
& + Q_1 d_2^c H_{19}^s V_{32}^s V_4 H_{35} \\
& + Q_2 d_1^c \Phi_{12} \bar{\Phi}'_4 V_9 V_{19} \\
& + Q_3 d_2^c N_3^c H_{30}^s V_{14} H_{35} \\
& + Q_1 d_3^c N_3^c \Phi_{12} H_{16}^s H_{26} V_{35} \\
& + Q_1 d_2^c N_3^c \Phi_{12} H_{20}^s H_{31}^s V_{31}^s \\
& + Q_1 d_2^c \Phi_{12} H_{18}^s H_{19}^s V_{15} V_{35} \\
& + Q_1 d_2^c \Phi_{12} H_{20}^s V_{31}^s H_{25} V_9 \\
& + Q_1 d_2^c \Phi_{12} H_{42} V_{13} H_{28} V_9 \\
& + Q_1 d_2^c \Phi_{13} V_2^s V_{11}^s V_{31}^s V_{32}^s \\
& + Q_1 d_2^c \Phi_{23} H_{29}^s H_{30}^s V_5 V_{17} \\
& + Q_1 d_2^c \bar{\Phi}'_4 H_{19}^s H_{32}^s V_1^s V_{32}^s \\
& + Q_1 d_3^c \Phi_{12} H_{15}^s H_{18}^s V_1^s V_{22}^s \\
& + Q_1 d_3^c \Phi_{12} H_{16}^s H_{38}^s V_5 V_{35} \\
& + Q_1 d_3^c \Phi_{12} H_{18}^s H_{29}^s V_{15} V_{27} \\
& + Q_1 d_3^c \Phi_{12} V_4 H_{35} H_{25} V_{30} \\
& + Q_1 d_3^c \Phi_{13} H_{37}^s V_{32}^s V_{14} H_{35} \\
& + Q_1 d_3^c \Phi_{23} H_{29}^s H_{30}^s V_2^s V_{21}^s \\
& + Q_1 d_3^c \bar{\Phi}'_5 H_{21}^s H_{31}^s H_{32}^s H_{39}^s \\
& + Q_2 d_1^c N_2^c \Phi_{23} H_{30}^s H_{32}^s V_1^s \\
& + Q_2 d_2^c \Phi_{13} V_1^s V_{12}^s V_{31}^s V_{32}^s \\
& + Q_2 d_2^c \Phi_{23} H_{29}^s H_{30}^s V_7 V_{15} \\
& + Q_2 d_2^c N_3^c \Phi_{23} H_{30}^s H_{26} V_{37} \\
& + Q_2 d_2^c \Phi_{23} H_{15}^s H_{30}^s V_{14} V_3 \\
& + Q_2 d_2^c \Phi_{23} H_{19}^s H_{31}^s V_{15} V_{37} \\
& + Q_2 d_2^c \Phi_{23} H_{21}^s H_{30}^s V_{19} V_{39} \\
& + Q_2 d_2^c \Phi_{23} H_{30}^s V_{31}^s H_{23} V_{35} \\
& + Q_2 d_2^c \Phi_{23} N_2^c \Phi_{23} H_{30}^s H_{32}^s V_{21}^s \\
& + Q_2 d_2^c N_3^c \bar{\Phi}'_5 H_{30}^s V_{14} H_{35} \\
& + Q_2 d_3^c \Phi_{12} H_{16}^s H_{38}^s V_{15} V_{35} \\
& + Q_2 d_3^c \Phi_{13} V_{31}^s V_{32}^s V_{24} V_{13} \\
& + Q_2 d_3^c \Phi_{23} H_{15}^s H_{30}^s V_7 V_{25} \\
& + Q_2 d_3^c \bar{\Phi}'_5 H_{16}^s H_{21}^s H_{32}^s H_{39}^s \\
& + Q_3 d_1^c N_2^c \Phi_{23} H_{30}^s H_{28} V_{39} \\
& + Q_3 d_1^c \Phi_{12} V_{22}^s V_{31}^s V_{34} V_3 \\
& + Q_3 d_1^c \Phi_{13} V_1^s V_{22}^s V_{31}^s V_{32}^s \\
& + Q_3 d_1^c \bar{\Phi}'_5 H_{19}^s H_{31}^s H_{42} H_{35} \\
& + Q_1 d_3^c H_{15}^s V_{32}^s H_{26} V_7 \\
& + Q_3 d_1^c N_3^c H_{30}^s H_{32}^s V_1^s \\
& + Q_3 d_2^c H_{30}^s H_{39}^s V_2^s V_{31}^s \\
& + Q_1 d_1^c \Phi_{12} H_{19}^s H_{31}^s V_{15} V_{35} \\
& + Q_1 d_2^c N_2^c \Phi_{12} H_{18}^s H_{20}^s V_{31}^s \\
& + Q_1 d_2^c \Phi_{12} H_{20}^s H_{21}^s V_1^s V_{12}^s \\
& + Q_1 d_2^c \Phi_{12} H_{39}^s V_{12}^s H_{28} V_9 \\
& + Q_1 d_2^c \Phi_{12} V_9 V_{39} V_{17} V_{37} \\
& + Q_1 d_2^c \Phi_{13} V_{31}^s V_{32}^s V_5 V_{17} \\
& + Q_1 d_2^c \Phi_{23} H_{29}^s V_{32}^s H_{28} V_{30} \\
& + Q_1 d_3^c N_1^c \Phi_{12} H_{16}^s H_{26} V_{35} \\
& + Q_1 d_3^c \Phi_{12} H_{15}^s H_{31}^s V_{11}^s V_{22}^s \\
& + Q_1 d_3^c \Phi_{12} H_{16}^s H_{42} V_4 V_{34} \\
& + Q_1 d_3^c \Phi_{12} H_{31}^s H_{38}^s V_{15} V_{35} \\
& + Q_1 d_3^c \Phi_{12} V_4 H_{35} H_{23} V_{27} \\
& + Q_1 d_3^c \Phi_{13} V_2^s V_{21}^s V_{31}^s V_{32}^s \\
& + Q_1 d_3^c \bar{\Phi}'_5 H_{29}^s H_{30}^s V_9 V_{29} \\
& + Q_1 d_3^c \bar{\Phi}'_5 H_{21}^s H_{31}^s H_{42} H_{35} \\
& + Q_2 d_1^c \Phi_{12} H_{16}^s H_{19}^s V_{15} V_{35} \\
& + Q_2 d_1^c \Phi_{13} V_{31}^s V_{32}^s V_7 V_{15} \\
& + Q_2 d_2^c N_1^c \Phi_{12} H_{16}^s H_{20}^s V_{31}^s \\
& + Q_2 d_2^c N_3^c \Phi_{23} H_{30}^s H_{32}^s V_2^s \\
& + Q_2 d_2^c \Phi_{23} H_{17}^s H_{30}^s V_4 V_{13} \\
& + Q_2 d_2^c \Phi_{23} H_{19}^s H_{32}^s V_{34} V_{13} \\
& + Q_2 d_2^c \Phi_{23} H_{30}^s H_{32}^s H_{37}^s H_{39}^s \\
& + Q_2 d_2^c \Phi_{23} H_{37}^s V_{32}^s H_{28} V_{40} \\
& + Q_2 d_3^c N_2^c \Phi_{23} H_{30}^s V_{24} H_{35} \\
& + Q_2 d_3^c \Phi_{12} H_{15}^s H_{16}^s V_{11}^s V_{22}^s \\
& + Q_2 d_3^c \Phi_{12} H_{18}^s H_{37}^s V_{19} V_{40} \\
& + Q_2 d_3^c \Phi_{23} H_{15}^s H_{22}^s H_{30}^s H_{37}^s \\
& + Q_2 d_3^c \Phi_{23} H_{29}^s H_{30}^s V_{12}^s V_{21}^s \\
& + Q_2 d_3^c \bar{\Phi}'_5 H_{16}^s H_{21}^s H_{42} H_{35} \\
& + Q_3 d_1^c N_3^c \Phi_{13} H_{16}^s H_{20}^s V_{31}^s \\
& + Q_3 d_1^c \Phi_{12} V_{30}^s V_{39} V_7 V_{37} \\
& + Q_3 d_1^c \Phi_{23} H_{29}^s H_{30}^s V_1^s V_{22}^s \\
& + Q_3 d_2^c N_1^c \Phi_{23} H_{30}^s H_{28} V_{39}
\end{aligned}$$

Possible down–quark mass terms from $\langle h_3 \rangle$ continued:

$$\begin{aligned}
& +Q_3 d_2^c N_3^c \overline{\Phi}_4' H_{22}^s H_{30}^s V_{31}^s & +Q_3 d_2^c \Phi_{12} H_{19}^s H_{20}^s V_{12}^s V_{21}^s & +Q_3 d_2^c \Phi_{12} H_{19}^s H_{20}^s V_{24} V_{13} \\
& +Q_3 d_2^c \Phi_{12} H_{19}^s H_{36}^s V_4 V_{13} & +Q_3 d_2^c \Phi_{12} H_{19}^s H_{36}^s V_9 V_{19} & +Q_3 d_2^c \Phi_{12} H_{20}^s H_{38}^s V_1^s V_{12}^s \\
& +Q_3 d_2^c \Phi_{12} H_{20}^s H_{38}^s V_7 V_{15} & +Q_3 d_2^c \Phi_{12} H_{20}^s V_{31}^s H_{25} V_{30} & +Q_3 d_2^c \Phi_{12} H_{20}^s V_{31}^s H_{23} V_{27} \\
& +Q_3 d_2^c \Phi_{12} V_{20} V_{39} V_{27} V_{37} & +Q_3 d_2^c \Phi_{12} V_{20} V_{40} V_{27} V_{35} & +Q_3 d_2^c \Phi_{12} V_{30} V_{39} V_{17} V_{37} \\
& +Q_3 d_2^c \Phi_{12} V_{30} V_{40} V_{17} V_{35} & +Q_3 d_2^c \Phi_{13} V_{11}^s V_{22}^s V_{31}^s V_{32}^s & +Q_3 d_2^c \Phi_{13} V_{31}^s V_{32}^s V_{14} V_{23} \\
& +Q_3 d_2^c \Phi_{23} H_{17}^s H_{20}^s H_{30}^s H_{37}^s & +Q_3 d_2^c \Phi_{23} H_{17}^s H_{30}^s V_4 V_{23} & +Q_3 d_2^c \Phi_{23} H_{17}^s H_{30}^s V_5 V_{27} \\
& +Q_3 d_2^c \Phi_{23} H_{19}^s H_{30}^s V_{25} V_{35} & +Q_3 d_2^c \Phi_{23} H_{19}^s H_{32}^s V_{34} V_{23} & +Q_3 d_2^c \Phi_{23} H_{19}^s V_{22}^s V_{34} H_{35} \\
& +Q_3 d_2^c \Phi_{23} H_{29}^s H_{30}^s V_{11}^s V_{22}^s & +Q_3 d_2^c \Phi_{23} H_{29}^s H_{30}^s V_{14} V_{23} & +Q_3 d_2^c \Phi_{23} H_{30}^s H_{39}^s V_4 V_{33} \\
& +Q_3 d_2^c \overline{\Phi}_4' H_{30}^s V_{31}^s H_{42} V_3 & +Q_3 d_2^c \overline{\Phi}_{56} H_{17}^s V_{32}^s H_{28} V_{20} & +Q_3 d_2^c \overline{\Phi}_{56} H_{17}^s V_{32}^s H_{26} V_{17} \\
& +Q_3 d_2^c \overline{\Phi}_{56} H_{18}^s H_{19}^s H_{32}^s H_{39}^s & +Q_3 d_2^c \overline{\Phi}_{56} H_{18}^s H_{19}^s H_{42} H_{35} & +\dots]
\end{aligned}$$

Possible electron mass terms from $\langle h_3 \rangle$:

$$\begin{aligned}
& h_3 [L_3 e_3^c \\
& + L_2 e_2^c V_{31}^s V_{32}^s \\
& + L_1 e_2^c \Phi_{12} V_4 V_{13} \\
& + L_1 e_2^c \Phi_{12} \overline{\Phi}_4' V_1^s V_{12}^s \\
& + L_2 e_1^c \Phi_{12} \Phi_4' V_2^s V_{11}^s \\
& + L_2 e_3^c N_3^c H_{30}^s H_{32}^s V_{11}^s \\
& + L_1 e_3^c N_2^c \Phi_{12} H_{20}^s H_{31}^s V_{31}^s \\
& + L_1 e_1^c \Phi_{12} H_{37}^s V_{32}^s H_{28} V_{39} \\
& + L_1 e_2^c \Phi_{13} V_{31}^s V_{32}^s V_4 V_{13} \\
& + L_1 e_2^c \Phi_{23} H_{29}^s H_{30}^s V_9 V_{19} \\
& + L_1 e_3^c \Phi_{12} V_2^s V_{31}^s V_{34} V_{23} \\
& + L_1 e_3^c \Phi_{13} H_{20}^s H_{37}^s V_{31}^s V_{32}^s \\
& + L_1 e_3^c \Phi_{23} H_{20}^s H_{29}^s H_{30}^s H_{37}^s \\
& + L_1 e_3^c \overline{\Phi}_{56} H_{29}^s V_{32}^s H_{28} V_{20} \\
& + L_2 e_1^c N_3^c \Phi_{23} H_{31}^s H_{28} V_{40} \\
& + L_2 e_1^c \Phi_{12} H_{19}^s H_{22}^s V_9 V_{19} \\
& + L_2 e_1^c \Phi_{12} H_{31}^s V_{31}^s H_{42} V_{23} \\
& + L_2 e_1^c \Phi_{12} H_{42} V_{13} H_{26} V_7 \\
& + L_2 e_1^c \Phi_{13} V_{31}^s V_{32}^s V_{14} V_3 \\
& + L_2 e_2^c N_1^c \Phi_{12} H_{16}^s H_{20}^s V_{31}^s \\
& + L_2 e_2^c N_3^c \Phi_{23} H_{30}^s H_{32}^s V_{21}^s \\
& + L_2 e_2^c \Phi_{23} H_{17}^s H_{30}^s V_4 V_{13} \\
& + L_2 e_2^c \Phi_{23} H_{19}^s H_{32}^s V_{34} V_{13} \\
& + L_2 e_2^c \Phi_{23} H_{30}^s H_{32}^s H_{37}^s H_{39}^s \\
& + L_2 e_2^c \Phi_{23} H_{37}^s V_{32}^s H_{28} V_{40} \\
& + L_2 e_3^c \Phi_{12} H_{19}^s H_{20}^s V_{12}^s V_{21}^s \\
& + L_2 e_3^c \Phi_{12} H_{19}^s H_{36}^s V_9 V_{19} \\
& + L_2 e_3^c \Phi_{12} H_{20}^s V_{31}^s H_{25} V_{30} \\
& + L_2 e_3^c \Phi_{12} V_{20} V_{40} V_{27} V_{35} \\
& + L_2 e_3^c \Phi_{13} V_{11}^s V_{22}^s V_{31}^s V_{32}^s \\
& + L_2 e_3^c \Phi_{23} H_{17}^s H_{30}^s V_4 V_{23} \\
& + L_2 e_3^c \Phi_{23} H_{19}^s H_{32}^s V_{34} V_{23} \\
& + L_2 e_3^c \Phi_{23} H_{29}^s H_{30}^s V_{14} V_{23} \\
& + L_2 e_3^c \overline{\Phi}_{56} H_{17}^s V_{32}^s H_{28} V_{20} \\
& + L_2 e_3^c \overline{\Phi}_{56} H_{18}^s H_{19}^s H_{42} H_{35} \\
& + L_3 e_1^c N_2^c \Phi_{23} H_{31}^s H_{28} V_{40} \\
& + L_3 e_1^c \Phi_{12} H_{16}^s H_{17}^s H_{20}^s H_{37}^s \\
& + L_3 e_1^c \Phi_{12} H_{16}^s H_{19}^s V_{25} V_{35} \\
& + L_3 e_1^c \Phi_{12} H_{16}^s H_{39}^s V_4 V_{33} \\
& + L_3 e_1^c \Phi_{12} H_{31}^s H_{38}^s V_{19} V_{39} \\
& + L_3 e_1^c \Phi_{12} V_1^s V_{32}^s V_{24} V_{33} \\
& + L_1 e_2^c \Phi_{12} V_9 V_{19} \\
& + L_1 e_2^c \Phi_{12} \overline{\Phi}_4' V_7 V_{15} \\
& + L_2 e_1^c \Phi_{12} \Phi_4' V_5 V_{17} \\
& + L_2 e_3^c N_3^c H_{30}^s V_{14} H_{35} \\
& + L_1 e_1^c N_3^c \Phi_{12} H_{16}^s H_{26} V_{35} \\
& + L_1 e_2^c N_2^c \Phi_{23} H_{30}^s V_4 H_{35} \\
& + L_1 e_2^c \Phi_{13} V_{31}^s V_{32}^s V_9 V_{19} \\
& + L_1 e_3^c N_2^c \Phi_{23} H_{30}^s H_{26} V_{35} \\
& + L_1 e_3^c \Phi_{12} V_9 V_{39} V_{27} V_{37} \\
& + L_1 e_3^c \Phi_{13} V_{31}^s V_{32}^s V_4 V_{23} \\
& + L_1 e_3^c \Phi_{23} H_{29}^s H_{30}^s V_4 V_{23} \\
& + L_1 e_3^c \overline{\Phi}_{56} H_{29}^s V_{32}^s H_{26} V_{17} \\
& + L_2 e_1^c \Phi_{12} H_{18}^s H_{19}^s V_{19} V_{39} \\
& + L_2 e_1^c \Phi_{12} H_{20}^s V_{31}^s H_{23} V_7 \\
& + L_2 e_1^c \Phi_{12} H_{39}^s V_{12}^s H_{26} V_7 \\
& + L_2 e_1^c \Phi_{12} V_{20} V_{39} V_7 V_{37} \\
& + L_2 e_1^c \Phi_{23} H_{29}^s H_{30}^s V_{14} V_3 \\
& + L_2 e_2^c N_1^c \Phi_{23} H_{30}^s H_{32}^s V_1^s \\
& + L_2 e_2^c N_3^c \Phi_{23} H_{30}^s V_{24} H_{35} \\
& + L_2 e_2^c \Phi_{23} H_{17}^s H_{30}^s V_9 V_{19} \\
& + L_2 e_2^c \Phi_{23} H_{19}^s V_{12}^s V_{34} H_{35} \\
& + L_2 e_2^c \Phi_{23} H_{30}^s H_{37}^s H_{42} H_{35} \\
& + L_2 e_3^c N_1^c \Phi_{23} H_{30}^s H_{28} V_{39} \\
& + L_2 e_3^c \Phi_{12} H_{19}^s H_{20}^s V_{24} V_{13} \\
& + L_2 e_3^c \Phi_{12} H_{20}^s H_{38}^s V_1^s V_{12}^s \\
& + L_2 e_3^c \Phi_{12} H_{20}^s V_{31}^s H_{23} V_{27} \\
& + L_2 e_3^c \Phi_{12} V_{30} V_{39} V_{17} V_{37} \\
& + L_2 e_3^c \Phi_{13} V_{31}^s V_{32}^s V_{14} V_{23} \\
& + L_2 e_3^c \Phi_{23} H_{17}^s H_{30}^s V_5 V_{27} \\
& + L_2 e_3^c \Phi_{23} H_{19}^s V_{22}^s V_{34} H_{35} \\
& + L_2 e_3^c \Phi_{23} H_{30}^s H_{39}^s V_4 V_{33} \\
& + L_2 e_3^c \overline{\Phi}_{56} H_{17}^s V_{32}^s H_{26} V_{17} \\
& + L_3 e_1^c N_1^c \Phi_{12} H_{16}^s H_{28} V_{39} \\
& + L_3 e_1^c N_3^c \Phi_{13} H_{16}^s H_{32}^s V_{11}^s \\
& + L_3 e_1^c \Phi_{12} H_{16}^s H_{17}^s V_4 V_{23} \\
& + L_3 e_1^c \Phi_{12} H_{16}^s H_{29}^s V_{11}^s V_{22}^s \\
& + L_3 e_1^c \Phi_{12} H_{17}^s H_{31}^s V_{19} V_{30} \\
& + L_3 e_1^c \Phi_{12} H_{32}^s V_1^s H_{25} V_{30} \\
& + L_3 e_1^c \Phi_{13} H_{16}^s H_{39}^s V_2^s V_{31}^s \\
& + L_2 e_1^c \Phi_{12} V_{14} V_3 \\
& + L_1 e_3^c N_3^c H_{30}^s V_4 H_{35} \\
& + L_2 e_1^c H_{19}^s H_{32}^s V_1^s V_{32}^s \\
& + L_2 e_2^c H_{30}^s H_{39}^s V_2^s V_{31}^s \\
& + L_1 e_1^c \Phi_{12} H_{19}^s H_{31}^s V_{15} V_{35} \\
& + L_1 e_2^c \Phi_{12} V_{22}^s V_{31}^s V_{34} V_{13} \\
& + L_1 e_2^c \Phi_{23} H_{29}^s H_{30}^s V_4 V_{13} \\
& + L_1 e_3^c N_3^c \overline{\Phi}_4 H_{30}^s H_{32}^s V_1^s \\
& + L_1 e_3^c \Phi_{12} V_9 V_{40} V_{27} V_{35} \\
& + L_1 e_3^c \Phi_{13} V_{31}^s V_{32}^s V_5 V_{27} \\
& + L_1 e_3^c \Phi_{23} H_{29}^s H_{30}^s V_5 V_{27} \\
& + L_2 e_1^c N_3^c \Phi_{12} H_{31}^s H_{36}^s V_{33} \\
& + L_2 e_1^c \Phi_{12} V_{20} V_{39} V_{27} V_{33} \\
& + L_2 e_3^c \Phi_{12} V_{30} V_{40} V_{17} V_{35} \\
& + L_2 e_3^c \Phi_{23} H_{17}^s H_{20}^s H_{30}^s H_{37}^s \\
& + L_2 e_3^c \overline{\Phi}_4 H_{19}^s H_{31}^s V_{25} V_{35} \\
& + L_2 e_3^c \Phi_{23} H_{29}^s H_{30}^s V_{11}^s V_{22}^s \\
& + L_2 e_3^c \overline{\Phi}_4 H_{30}^s V_{31}^s H_{42} V_3 \\
& + L_2 e_3^c \overline{\Phi}_{56} H_{18}^s H_{19}^s H_{32}^s H_{39}^s \\
& + L_3 e_1^c N_2^c \Phi_{12} H_{31}^s H_{36}^s V_{31}^s \\
& + L_3 e_1^c N_3^c \Phi_{13} H_{16}^s V_{14} H_{35} \\
& + L_3 e_1^c \Phi_{12} H_{16}^s H_{17}^s V_5 V_{27} \\
& + L_3 e_1^c \Phi_{12} H_{16}^s H_{29}^s V_{14} V_{23} \\
& + L_3 e_1^c \Phi_{12} H_{17}^s H_{31}^s V_{15} V_{27} \\
& + L_3 e_1^c \Phi_{12} H_{32}^s V_1^s H_{23} V_{27} \\
& + L_3 e_1^c \Phi_{13} H_{22}^s H_{37}^s V_{31}^s V_{35}^s
\end{aligned}$$

Possible electron mass terms from $\langle h_3 \rangle$ continued:

$$\begin{aligned}
& + L_3 e_1^c \Phi_{13} H_{31}^s H_{39}^s V_{12}^s V_{31}^s & + L_3 e_1^c \Phi_{13} H_{31}^s V_{31}^s H_{42} V_{13} & + L_3 e_1^c \Phi_{13} V_{31}^s V_{32}^s V_{24} V_3 \\
& + L_3 e_1^c \Phi_{13} V_{31}^s V_{32}^s V_7 V_{25} & + L_3 e_1^c \Phi_{23} H_{22}^s H_{29}^s H_{30}^s H_{37}^s & + L_3 e_1^c \Phi_{23} H_{29}^s H_{30}^s V_{24} V_3 \\
& + L_3 e_1^c \Phi_{23} H_{29}^s H_{30}^s V_7 V_{25} & + L_3 e_1^c \Phi_4 H_{15}^s V_{32}^s H_{26} V_7 & + L_3 e_2^c N_2^c \Phi_{23} H_{16}^s H_{26} V_{37} \\
& + L_3 e_2^c N_2^c \Phi_{23} H_{30}^s H_{32}^s V_{21} & + L_3 e_2^c N_2^c \Phi_{23} H_{30}^s V_{24} H_{35} & + L_3 e_2^c N_3^c \overline{\Phi}_{56}^s H_{30}^s H_{32}^s V_{11} \\
& + L_3 e_2^c N_3^c \overline{\Phi}_{56}^s H_{30}^s V_{14} H_{35} & + L_3 e_2^c \Phi_{12} H_{15}^s H_{16}^s V_{11}^s V_{22} & + L_3 e_2^c \Phi_{12} H_{15}^s H_{16}^s V_{14} V_{23} \\
& + L_3 e_2^c \Phi_{12} H_{16}^s H_{38}^s V_{15} V_{35} & + L_3 e_2^c \Phi_{12} H_{18}^s H_{37}^s V_{19} V_{40} & + L_3 e_2^c \Phi_{13} V_{12}^s V_{21}^s V_{31}^s V_{32}^s \\
& + L_3 e_2^c \Phi_{13} V_{31}^s V_{32}^s V_{24} V_{13} & + L_3 e_2^c \Phi_{23} H_{15}^s H_{22}^s H_{30}^s H_{37}^s & + L_3 e_2^c \Phi_{23} H_{15}^s H_{30}^s V_{24} V_3 \\
& + L_3 e_2^c \Phi_{23} H_{15}^s H_{30}^s V_7 V_{25} & + L_3 e_2^c \Phi_{23} H_{29}^s H_{30}^s V_{12}^s V_{21} & + L_3 e_2^c \Phi_{23} H_{29}^s H_{30}^s V_{24} V_{13} \\
& + L_3 e_2^c \overline{\Phi}_{56}^s H_{16}^s H_{21}^s H_{32}^s H_{39}^s & + L_3 e_2^c \overline{\Phi}_{56}^s H_{16}^s H_{21}^s H_{42} H_{35} & + L_3 e_2^c \overline{\Phi}_{56}^s H_{30}^s H_{39}^s V_2^s V_{31}^s \\
& + \dots]
\end{aligned}$$

Possible down-quark mass terms from $\langle h_4 \equiv H_{41} \rangle$:

$$\begin{aligned}
& h_4 [Q_1 d_3^c H_{21}^s \\
& + Q_2 d_2^c \Phi_{23} H_{38}^s \\
& + Q_1 d_3^c H_{15}^s H_{18}^s H_{19}^s \\
& + Q_1 d_1^c N_1^c \Phi_{12} H_{26} V_7 \\
& + Q_1 d_1^c N_3^c \Phi_{13} H_{18}^s V_{31}^s \\
& + Q_1 d_1^c \Phi_{12} H_{21}^s V_{11}^s V_{22}^s \\
& + Q_1 d_1^c \Phi_{12} H_{38}^s V_5 V_7 \\
& + Q_1 d_2^c \Phi_{12} H_{21}^s V_{11}^s V_{22}^s \\
& + Q_1 d_2^c \Phi_{12} H_{38}^s V_5 V_{17} \\
& + Q_2 d_1^c \Phi_{12} H_{19}^s V_{24} V_{13} \\
& + Q_2 d_1^c \Phi_{12} V_{31}^s H_{25} V_{30} \\
& + Q_2 d_2^c \Phi_{12} H_{38}^s V_{11}^s V_{12}^s \\
& + Q_2 d_2^c \Phi_{12} H_{38}^s V_{15} V_{17} \\
& + Q_3 d_3^c \Phi_{13} H_{38}^s V_{19} V_{20} \\
& + Q_3 d_3^c \Phi_{23} H_{38}^s V_{34} V_{33} \\
& + Q_3 d_3^c \bar{\Phi}_{56} H_{21}^s H_{22}^s H_{38}^s \\
& + Q_1 d_1^c N_1^c \Phi_{12} \Phi_4 H_{28} V_9 \\
& + Q_1 d_1^c N_3^c H_{18}^s H_{29}^s H_{30}^s V_{31}^s \\
& + Q_1 d_1^c \Phi_{12} \Phi_4 H_{21}^s V_4 V_{23} \\
& + Q_1 d_1^c \Phi_{12} \bar{\Phi}_4 H_{19}^s V_{24} V_3 \\
& + Q_1 d_2^c N_1^c \Phi_{13} \Phi_{23} H_{28} V_{20} \\
& + Q_1 d_2^c N_3^c H_{21}^s H_{30}^s H_{32}^s V_{31}^s \\
& + Q_1 d_2^c \bar{\Phi}_4 H_{19}^s V_{20} V_{29} \\
& + Q_1 d_2^c \Phi_{12} \Phi_{56} H_{21}^s V_{24} V_{13} \\
& + Q_1 d_2^c \Phi_{13} \Phi_{23} H_{38}^s V_2^s V_{11}^s \\
& + Q_1 d_2^c H_{15}^s H_{19}^s V_{32}^s H_{28} V_{20} \\
& + Q_1 d_2^c H_{21}^s H_{30}^s H_{39}^s V_2^s V_{31}^s \\
& + Q_1 d_3^c \Phi_{13} \Phi_{23} H_{38}^s V_2^s V_{21}^s \\
& + Q_1 d_3^c \bar{\Phi}_4 H_{31}^s V_{32}^s H_{23} V_{17} \\
& + Q_1 d_3^c H_{15}^s H_{38}^s V_{32}^s H_{26} V_7 \\
& + Q_2 d_1^c \Phi_{12} \bar{\Phi}_4 H_{21}^s V_{19} V_{30} \\
& + Q_2 d_1^c \Phi_{12} \bar{\Phi}_{56} H_{19}^s V_{11}^s V_{22}^s \\
& + Q_2 d_1^c \Phi_{13} \Phi_{23} H_{19}^s V_{24} V_{13} \\
& + Q_2 d_1^c \Phi_{13} \Phi_{23} H_{31}^s H_{25} V_{30} \\
& + Q_2 d_1^c H_{17}^s H_{30}^s H_{37}^s V_{31}^s V_{32}^s \\
& + Q_2 d_3^c \Phi_{13} \Phi_{23} H_{38}^s V_{12}^s V_{21}^s \\
& + Q_2 d_3^c H_{15}^s H_{30}^s V_{31}^s H_{25} V_{20} \\
& + Q_3 d_1^c N_3^c H_{30}^s H_{32}^s H_{38}^s V_1^s \\
& + Q_3 d_2^c N_1^c H_{19}^s H_{30}^s V_{14} H_{35} \\
& + Q_3 d_2^c N_3^c H_{30}^s H_{38}^s V_{14} H_{35} \\
& + Q_3 d_3^c \Phi_{13} H_{38}^s V_{31}^s V_{32}^s V_2^s \\
& + Q_3 d_3^c \Phi_{12} H_{38}^s V_1^s V_{22}^s \\
& + Q_3 d_3^c N_1^c \Phi_{13} H_{38}^s V_1^s V_{22}^s \\
& + Q_3 d_3^c N_3^c H_{21}^s H_{30}^s H_{36}^s V_{31}^s \\
& + Q_3 d_3^c \Phi_{13} \Phi_{23} H_{38}^s V_{11}^s V_{22}^s \\
& + Q_3 d_4^c N_2^c \Phi_{13} H_{38}^s V_{12}^s V_{21}^s \\
& + Q_3 d_4^c \Phi_{13} \bar{\Phi}_4 H_{31}^s V_{25} V_{20} \\
& + Q_3 d_4^c \bar{\Phi}_4 \bar{\Phi}_4 H_{16}^s H_{17}^s H_{19}^s \\
& + Q_2 d_1^c \Phi_{12} \bar{\Phi}_{13} H_{17}^s H_{30}^s H_{37}^s \\
& + Q_2 d_1^c \Phi_{12} \bar{\Phi}_4 H_{21}^s V_{15} V_{27} \\
& + Q_2 d_1^c \Phi_{12} \bar{\Phi}'_4 H_{38}^s V_9 V_{19} \\
& + Q_2 d_1^c \Phi_{12} \bar{\Phi}_{56} H_{19}^s V_{14} V_{23} \\
& + Q_2 d_1^c \Phi_{13} \Phi_{23} H_{38}^s V_1^s V_{12}^s \\
& + Q_2 d_1^c \Phi_{13} \Phi_{23} V_{31}^s H_{23} V_{27} \\
& + Q_2 d_2^c H_{21}^s H_{30}^s H_{39}^s V_{12}^s V_{31}^s \\
& + Q_2 d_3^c \Phi_{13} \Phi_{23} H_{38}^s V_{24} V_{13} \\
& + Q_2 d_3^c H_{15}^s H_{30}^s V_{31}^s H_{23} V_{17} \\
& + Q_3 d_1^c \Phi_{13} \Phi_{23} H_{38}^s V_1^s V_{22}^s \\
& + Q_3 d_2^c N_1^c H_{19}^s H_{30}^s H_{32}^s V_{11}^s \\
& + Q_3 d_2^c N_3^c H_{30}^s H_{32}^s H_{38}^s V_{11}^s \\
& + Q_3 d_2^c \Phi_{13} \Phi_{23} H_{38}^s V_{14} V_{23}
\end{aligned}$$

Possible down–quark mass terms from $\langle h_4 \equiv H_{41} \rangle$ continued:

$$\begin{aligned}
& +Q_3 d_2^c \bar{\Phi}_{13} \bar{\Phi}_4' H_{17}^s H_{19}^s H_{30}^s \\
& +Q_3 d_2^c H_{21}^s H_{30}^s H_{39}^s V_{22}^s V_{31}^s \\
& +Q_3 d_3^c N_1^c H_{16}^s H_{19}^s H_{26}^s V_{37} \\
& +Q_3 d_3^c N_1^c H_{30}^s H_{32}^s H_{38}^s V_1^s \\
& +Q_3 d_3^c N_2^c H_{30}^s H_{32}^s H_{38}^s V_1^s \\
& +Q_3 d_3^c N_3^c H_{30}^s H_{38}^s V_{14}^s H_{35} \\
& +Q_3 d_3^c N_3^c H_{30}^s H_{38}^s V_{24}^s H_{35} \\
& +Q_3 d_3^c H_{15}^s H_{16}^s H_{19}^s H_{20}^s H_{38}^s \\
& +Q_3 d_3^c H_{15}^s H_{30}^s H_{38}^s V_{14}^s V_3 \\
& +Q_3 d_3^c H_{17}^s H_{21}^s H_{30}^s V_{15}^s V_{27} \\
& +Q_3 d_3^c H_{19}^s H_{31}^s H_{38}^s V_{15}^s V_{37} \\
& +Q_3 d_3^c H_{21}^s H_{30}^s H_{38}^s V_{19}^s V_{39} \\
& +Q_3 d_3^c H_{29}^s H_{30}^s H_{38}^s V_{14}^s V_{13} \\
& +Q_3 d_3^c H_{30}^s H_{32}^s H_{37}^s H_{38}^s H_{39}^s \\
& +Q_3 d_3^c H_{37}^s H_{38}^s V_{32}^s H_{28}^s V_{40}^s \\
& +Q_3 d_2^c H_{18}^s H_{19}^s H_{30}^s V_{15}^s V_{37} \\
& +Q_3 d_2^c H_{21}^s H_{30}^s V_{31}^s H_{42}^s V_{23} \\
& +Q_3 d_3^c N_1^c H_{19}^s H_{30}^s H_{32}^s V_{21}^s \\
& +Q_3 d_3^c N_2^c H_{21}^s H_{30}^s H_{36}^s V_{31}^s \\
& +Q_3 d_3^c N_3^c H_{16}^s H_{38}^s H_{26}^s V_{37} \\
& +Q_3 d_3^c \bar{\Phi}_4 \bar{\Phi}_{56} H_{21}^s H_{21}^s H_{36}^s \\
& +Q_3 d_3^c H_{15}^s H_{19}^s H_{30}^s V_{20}^s V_{29} \\
& +Q_3 d_3^c H_{16}^s H_{19}^s H_{38}^s V_5^s V_{37} \\
& +Q_3 d_3^c H_{17}^s H_{30}^s H_{38}^s V_4^s V_{13} \\
& +Q_3 d_3^c H_{19}^s H_{32}^s H_{38}^s V_{34}^s V_{13} \\
& +Q_3 d_3^c H_{21}^s V_{30}^s V_{40}^s V_7^s V_{37} \\
& +Q_3 d_3^c H_{29}^s H_{30}^s H_{38}^s V_{19}^s V_{20} \\
& +Q_3 d_3^c H_{30}^s H_{37}^s H_{38}^s H_{42}^s H_{35} \\
& +Q_3 d_3^c H_{38}^s V_{31}^s V_{32}^s V_{34}^s V_{33} \\
& +Q_3 d_2^c H_{19}^s V_9^s V_{40}^s V_{17}^s V_{37} \\
& +Q_3 d_2^c H_{30}^s H_{38}^s H_{39}^s V_2^s V_{31}^s \\
& +Q_3 d_3^c N_1^c H_{19}^s H_{30}^s V_{24}^s H_{35} \\
& +Q_3 d_3^c N_2^c H_{30}^s H_{32}^s H_{38}^s V_{11}^s \\
& +Q_3 d_3^c N_3^c H_{30}^s H_{32}^s H_{38}^s V_{21}^s \\
& +Q_3 d_3^c \bar{\Phi}_4 \bar{\Phi}_{56} H_{19}^s H_{19}^s H_{36}^s \\
& +Q_3 d_3^c H_{15}^s H_{19}^s H_{30}^s V_{17}^s V_{25} \\
& +Q_3 d_3^c H_{17}^s H_{21}^s H_{30}^s V_{19}^s V_{30} \\
& +Q_3 d_3^c H_{17}^s H_{30}^s H_{38}^s V_9^s V_{19} \\
& +Q_3 d_3^c H_{19}^s H_{38}^s V_{12}^s V_{34}^s H_{35} \\
& +Q_3 d_3^c H_{29}^s H_{30}^s H_{38}^s V_{11}^s V_{12} \\
& +Q_3 d_3^c H_{29}^s H_{30}^s H_{38}^s V_{15}^s V_{17} \\
& +Q_3 d_3^c H_{30}^s H_{38}^s V_{31}^s H_{23}^s V_{35} \\
& +Q_3 d_3^c H_{38}^s V_9^s V_{40}^s V_7^s V_{37} \\
& +\dots
\end{aligned}$$

Possible electron mass terms from $\langle h_4 \equiv H_{41} \rangle$:

$$\begin{aligned}
& h_4 [L_1 e_3^c \bar{\Phi}_4 H_{19}^s \\
& + L_1 e_3^c \Phi_2 \bar{\Phi}_4 H_{19}^s \\
& + L_1 e_1^c N_1^c \Phi_{12} H_{26} V_7 \\
& + L_1 e_1^c N_3^c \Phi_{13} H_{18}^s V_{31} \\
& + L_1 e_1^c \Phi_{12} H_{21}^s V_1^s V_{22} \\
& + L_1 e_1^c \Phi_{12} H_{38}^s V_5 V_7 \\
& + L_1 e_2^c \Phi_{12} H_{38}^s V_4 V_{13} \\
& + L_2 e_1^c \Phi_{12} H_{19}^s V_{20} V_{29} \\
& + L_2 e_2^c \Phi_{12} H_{38}^s V_{11} V_{12} \\
& + L_2 e_2^c \Phi_{12} H_{38}^s V_{15} V_{17} \\
& + L_3 e_1^c \Phi_{13} V_{31}^s H_{23} V_{17} \\
& + L_3 e_3^c \Phi_{13} H_{38}^s V_{11} V_{12} \\
& + L_3 e_3^c \Phi_{13} H_{38}^s V_{15} V_{17} \\
& + L_3 e_3^c \bar{\Phi}_{23} H_{38}^s V_{31} V_{32} \\
& + L_3 e_3^c \bar{\Phi}'_{56} H_{15}^s H_{16}^s H_{38}^s \\
& + L_1 e_1^c N_1^c \Phi_{12} \Phi_4 H_{28} V_9 \\
& + L_1 e_1^c N_3^c H_{18}^s H_{29}^s H_{30}^s V_{31}^s \\
& + L_1 e_1^c \Phi_{12} \Phi_4 H_{21}^s V_4 V_{23} \\
& + L_1 e_1^c \Phi_{12} \bar{\Phi}_4 H_{19}^s V_{24} V_3 \\
& + L_1 e_2^c N_3^c H_{15}^s H_{18}^s H_{30}^s V_{31}^s \\
& + L_1 e_2^c \Phi_{12} \bar{\Phi}_4 H_{19}^s V_{12}^s V_{21}^s \\
& + L_1 e_2^c \Phi_{12} \bar{\Phi}_4 V_{31}^s H_{25} V_{30} \\
& + L_1 e_2^c \Phi_{13} \Phi_{23} H_{21}^s V_{19} V_{30} \\
& + L_1 e_2^c \Phi_{13} \Phi_{23} H_{38}^s V_9 V_{19} \\
& + L_1 e_3^c N_1^c \Phi_{13} \Phi_{23} H_{28} V_{30} \\
& + L_1 e_3^c N_2^c H_{20}^s H_{21}^s H_{30}^s V_{31}^s \\
& + L_1 e_3^c N_3^c H_{30}^s H_{38}^s V_4 H_{35} \\
& + L_1 e_3^c \Phi_{13} \Phi_{23} H_{38}^s V_5 V_{27} \\
& + L_1 e_3^c H_{15}^s H_{19}^s V_{32}^s H_{28} V_{30} \\
& + L_1 e_3^c H_{19}^s H_{37}^s V_{32}^s H_{26} V_{37} \\
& + L_2 e_1^c N_3^c H_{17}^s H_{18}^s H_{30}^s V_{31}^s \\
& + L_2 e_1^c \Phi_{12} \bar{\Phi}_4 H_{21}^s V_{11}^s V_{22}^s \\
& + L_2 e_1^c \Phi_{12} \Phi_{56} H_{19}^s V_{19} V_{30} \\
& + L_2 e_1^c \Phi_{13} \Phi_{23} H_{19}^s V_{17} V_{25} \\
& + L_2 e_1^c H_{19}^s H_{19}^s H_{32}^s V_{21}^s V_{32}^s \\
& + L_2 e_1^c H_{21}^s H_{30}^s V_{31}^s H_{42} V_3 \\
& + L_2 e_3^c N_1^c H_{19}^s H_{30}^s H_{32}^s V_{11}^s \\
& + L_2 e_3^c N_3^c H_{30}^s H_{32}^s H_{38}^s V_{11}^s \\
& + L_2 e_3^c \Phi_{13} \Phi_{23} H_{38}^s V_{14} V_{23} \\
& + L_2 e_3^c H_{19}^s V_9 V_{40} V_{17} V_{37} \\
& + L_2 e_3^c H_{30}^s H_{38}^s H_{39}^s V_2^s V_{31}^s \\
& + L_2 e_3^c \Phi_{13} \Phi_4 H_{18}^s V_{31}^s \\
& + L_2 e_2^c H_{38}^s V_{31}^s V_{32}^s \\
& + L_1 e_1^c N_2^c \Phi_{12} H_{28} V_{20} \\
& + L_1 e_1^c \Phi_{12} H_{19}^s V_2^s V_{21}^s \\
& + L_1 e_1^c \Phi_{12} H_{38}^s V_1^s V_2^s \\
& + L_1 e_1^c \Phi_{12} H_{21}^s V_{19} V_{30} \\
& + L_1 e_2^c \Phi_{12} H_{38}^s V_9 V_{19} \\
& + L_2 e_1^c \Phi_{12} H_{19}^s V_{17} V_{25} \\
& + L_2 e_2^c \Phi_{12} H_{38}^s V_{14} V_{13} \\
& + L_2 e_2^c \Phi_{12} H_{38}^s V_{19} V_{20} \\
& + L_3 e_1^c N_2^c \Phi_{56} H_{30}^s V_{31}^s \\
& + L_2 e_1^c \Phi_{12} H_{38}^s V_{14} V_3 \\
& + L_2 e_2^c \Phi_{12} H_{38}^s V_{19} V_{20} \\
& + L_3 e_1^c \Phi_{13} V_{31}^s H_{25} V_{20} \\
& + L_3 e_1^c \bar{\Phi}_4 H_{16}^s H_{17}^s H_{19}^s \\
& + L_3 e_3^c \Phi_{13} H_{38}^s V_{19} V_{20} \\
& + L_3 e_3^c \bar{\Phi}_{23} H_{38}^s V_{34} V_{33} \\
& + L_3 e_3^c \bar{\Phi}_{56} H_{21}^s H_{22}^s H_{38}^s \\
& + L_3 e_3^c \bar{\Phi}'_{56} H_{19}^s H_{20}^s H_{38}^s \\
& + L_1 e_1^c N_2^c \Phi_{13} \Phi_{23} H_{26} V_{17} \\
& + L_1 e_1^c \Phi_{12} \Phi_4 H_{20}^s H_{21}^s H_{37}^s \\
& + L_1 e_1^c \bar{\Phi}_4 H_{19}^s H_{22}^s H_{37}^s \\
& + L_1 e_1^c H_{15}^s H_{19}^s V_{32}^s H_{26} V_7 \\
& + L_1 e_2^c \Phi_{12} \bar{\Phi}_4 V_{31}^s H_{23} V_{27} \\
& + L_1 e_2^c \Phi_{12} \bar{\Phi}_4 H_{38}^s V_7 V_{15} \\
& + L_1 e_2^c \Phi_{12} \bar{\Phi}_4 H_{38}^s V_{17} V_{25} \\
& + L_1 e_2^c \Phi_{13} \Phi_{23} H_{38}^s V_4 V_{13} \\
& + L_1 e_2^c \Phi_{23} \bar{\Phi}_{56} H_{18}^s H_{29}^s H_{38}^s \\
& + L_1 e_3^c N_1^c H_{19}^s H_{30}^s V_4 H_{35} \\
& + L_1 e_3^c N_3^c H_{15}^s H_{30}^s H_{26} V_{17} \\
& + L_1 e_3^c \Phi_{13} \Phi_{23} H_{38}^s V_4 V_{23} \\
& + L_1 e_3^c H_{15}^s H_{19}^s H_{30}^s V_5 V_{17} \\
& + L_1 e_3^c H_{19}^s H_{21}^s H_{30}^s V_{15} V_{35} \\
& + L_1 e_3^c \Phi_{13} \Phi_{23} H_{38}^s V_4 V_{23} \\
& + L_1 e_3^c H_{15}^s H_{19}^s H_{30}^s V_5 V_{17} \\
& + L_1 e_3^c H_{19}^s H_{21}^s H_{30}^s V_{15} V_{35} \\
& + L_2 e_1^c N_1^c \Phi_{12} \bar{\Phi}_4 H_{26} V_{17} \\
& + L_2 e_1^c \Phi_{12} \Phi_4 H_{21}^s V_{14} V_{23} \\
& + L_2 e_1^c \Phi_{12} \bar{\Phi}_4 H_{38}^s V_5 V_{17} \\
& + L_2 e_1^c \Phi_{13} \Phi_{23} H_{19}^s V_{20} V_{29} \\
& + L_2 e_1^c \Phi_{23} \bar{\Phi}'_{56} H_{16}^s H_{29}^s H_{38}^s \\
& + L_2 e_1^c H_{19}^s H_{32}^s H_{38}^s V_1^s V_{32}^s \\
& + L_2 e_2^c H_{21}^s H_{30}^s V_{31}^s H_{42} V_{13} \\
& + L_2 e_3^c N_3^c H_{21}^s H_{30}^s H_{36}^s V_{31}^s \\
& + L_2 e_3^c \Phi_{13} \Phi_{23} H_{38}^s V_{11}^s V_{22}^s \\
& + L_2 e_3^c H_{18}^s H_{19}^s H_{19}^s V_{15} V_{37} \\
& + L_2 e_3^c H_{21}^s H_{30}^s V_{31}^s H_{42} V_{23} \\
& + L_3 e_1^c N_1^c H_{17}^s H_{30}^s H_{31}^s V_3^s
\end{aligned}$$

Possible electron mass terms from $\langle h_4 \equiv H_{41} \rangle$ continued:

$$\begin{aligned}
& +L_3 e_1^c N_2^c H_{17}^s H_{18}^s H_{30}^s V_{31}^s & +L_3 e_1^c N_2^c H_{21}^s H_{22}^s H_{30}^s V_{31}^s & +L_3 e_1^c N_3^c H_{16}^s H_{21}^s H_{28}^s V_{40} \\
& +L_3 e_1^c \Phi_{12} \bar{\Phi}_{23} V_{31}^s H_{25}^s V_{20} & +L_3 e_1^c \Phi_{12} \bar{\Phi}_{23} V_{31}^s H_{23}^s V_{17} & +L_3 e_1^c \Phi_{13} \Phi_{23} H_{22}^s H_{37}^s H_{38}^s \\
& +L_3 e_1^c \Phi_{13} \Phi_{23} H_{38}^s V_{24}^s V_{3} & +L_3 e_1^c \Phi_{13} \Phi_{23} H_{38}^s V_7^s V_{25} & +L_3 e_1^c H_{17}^s H_{21}^s H_{30}^s V_1^s V_{12} \\
& +L_3 e_1^c H_{17}^s H_{21}^s H_{30}^s V_7^s V_{15} & +L_3 e_1^c H_{17}^s H_{30}^s V_{31}^s H_{25}^s V_9 & +L_3 e_1^c H_{19}^s H_{21}^s H_{31}^s V_{19}^s V_{40} \\
& +L_3 e_1^c H_{19}^s H_{21}^s H_{35}^s V_{13}^s V_{33} & +L_3 e_1^c H_{19}^s H_{31}^s V_{31}^s H_{23}^s V_{37} & +L_3 e_1^c H_{21}^s H_{30}^s V_{31}^s H_{25}^s V_{39} \\
& +L_3 e_1^c H_{29}^s H_{30}^s V_{31}^s H_{25}^s V_{20} & +L_3 e_1^c H_{29}^s H_{30}^s V_{31}^s H_{23}^s V_{17} & +L_3 e_1^c \Phi_{13} \Phi_{23} H_{38}^s V_{12}^s V_{21} \\
& +L_3 e_2^c \Phi_{13} \Phi_{23} H_{38}^s V_{24}^s V_{13} & +L_3 e_2^c \bar{\Phi}_{13} \Phi_4 H_{15}^s H_{21}^s H_{30}^s & +L_3 e_2^c H_{15}^s H_{30}^s V_{31}^s H_{25}^s V_{20} \\
& +L_3 e_2^c H_{15}^s H_{30}^s V_{31}^s H_{23}^s V_{17} & +L_3 e_2^c H_{16}^s H_{19}^s H_{21}^s V_{15}^s V_{37} & +L_3 e_3^c N_1^c H_{16}^s H_{19}^s H_{26}^s V_{37} \\
& +L_3 e_3^c N_1^c H_{19}^s H_{30}^s H_{32}^s V_{21} & +L_3 e_3^c N_1^c H_{19}^s H_{30}^s V_{24}^s H_{35} & +L_3 e_3^c N_1^c H_{30}^s H_{32}^s H_{38}^s V_1^s \\
& +L_3 e_3^c N_2^c H_{21}^s H_{30}^s H_{36}^s V_{31}^s & +L_3 e_3^c N_2^c H_{30}^s H_{32}^s H_{38}^s V_{11} & +L_3 e_3^c N_2^c H_{30}^s H_{38}^s V_{14}^s H_{35} \\
& +L_3 e_3^c N_3^c H_{16}^s H_{38}^s H_{26}^s V_{37} & +L_3 e_3^c N_3^c H_{30}^s H_{32}^s H_{38}^s V_{21} & +L_3 e_3^c N_3^c H_{30}^s H_{38}^s V_{24}^s H_{35} \\
& +L_3 e_3^c \Phi_4 \bar{\Phi}_{56} H_{21}^s H_{21}^s H_{36}^s & +L_3 e_3^c \bar{\Phi}_4 \bar{\Phi}_{56} H_{19}^s H_{19}^s H_{36}^s & +L_3 e_3^c H_{15}^s H_{16}^s H_{19}^s H_{20}^s H_{38}^s \\
& +L_3 e_3^c H_{15}^s H_{19}^s H_{30}^s V_{20}^s V_{29} & +L_3 e_3^c H_{15}^s H_{19}^s H_{30}^s V_{17}^s V_{25} & +L_3 e_3^c H_{15}^s H_{30}^s H_{38}^s V_{14}^s V_3 \\
& +L_3 e_3^c H_{16}^s H_{19}^s H_{38}^s V_5^s V_{37} & +L_3 e_3^c H_{17}^s H_{21}^s H_{30}^s V_{19}^s V_{30} & +L_3 e_3^c H_{17}^s H_{21}^s H_{30}^s V_{15}^s V_{27} \\
& +L_3 e_3^c H_{17}^s H_{30}^s H_{38}^s V_4^s V_{13} & +L_3 e_3^c H_{17}^s H_{30}^s H_{38}^s V_9^s V_{19} & +L_3 e_3^c H_{19}^s H_{31}^s H_{38}^s V_{15}^s V_{37} \\
& +L_3 e_3^c H_{19}^s H_{32}^s H_{38}^s V_{34}^s V_{13} & +L_3 e_3^c H_{19}^s H_{38}^s V_{12}^s V_{34}^s H_{35} & +L_3 e_3^c H_{21}^s H_{30}^s H_{38}^s V_{19}^s V_{39} \\
& +L_3 e_3^c H_{21}^s V_{30}^s V_{40}^s V_7^s V_{37} & +L_3 e_3^c H_{29}^s H_{30}^s H_{38}^s V_{11}^s V_{12} & +L_3 e_3^c H_{29}^s H_{30}^s H_{38}^s V_{14}^s V_{13} \\
& +L_3 e_3^c H_{29}^s H_{30}^s H_{38}^s V_{19}^s V_{20} & +L_3 e_3^c H_{29}^s H_{30}^s H_{38}^s V_{15}^s V_{17} & +L_3 e_3^c H_{30}^s H_{32}^s H_{37}^s H_{38}^s H_{39} \\
& +L_3 e_3^c H_{30}^s H_{37}^s H_{38}^s H_{42}^s H_{35} & +L_3 e_3^c H_{30}^s H_{38}^s V_{31}^s H_{23}^s V_{35} & +L_3 e_3^c H_{37}^s H_{38}^s V_{32}^s H_{28}^s V_{40} \\
& +L_3 e_3^c H_{38}^s V_{31}^s V_{32}^s V_{34}^s V_{33} & +L_3 e_3^c H_{38}^s V_9^s V_{40}^s V_7^s V_{37} & +\dots]
\end{aligned}$$

C Potential FNY Higgs Doublet Mass Terms (through 8th order)

Possible Higgs $h_1 \bar{h}_1$ mass terms (through 8th order):

$$\begin{aligned}
& h_1 \bar{h}_1 [N_3^c H_{30}^s H_{32}^s V_{21}^s + \Phi_{23} H_{29}^s H_{30}^s H_{36}^s H_{37}^s + \Phi_{23} H_{29}^s H_{30}^s V_{25} V_{27} \\
& + N_3^c H_{15}^s H_{30}^s H_{31}^s H_{36}^s V_{31}^s + \Phi_{23} \bar{\Phi}_{56} H_{25} H_{28} H_{23} H_{26} + H_{15}^s H_{30}^s H_{39} V_{12}^s H_{26} V_7 \\
& + H_{29}^s H_{30}^s V_{31}^s V_{32}^s V_{24} V_{23} + N_3^c H_{30}^s V_{24} H_{35} + \Phi_{23} H_{29}^s H_{30}^s V_{21}^s V_{22}^s \\
& + \bar{\Phi}_{56} H_{31}^s H_{32}^s H_{38}^s H_{39}^s + N_3^c H_{30}^s H_{31}^s V_{31}^s V_{25} V_{37} + \Phi_{23} \bar{\Phi}_{56} H_{29}^s H_{31}^s H_{36}^s H_{38}^s \\
& + \Phi_{23} \bar{\Phi}_{56} H_{28} H_{28} H_{23} H_{23} + H_{15}^s H_{30}^s H_{31}^s H_{39} V_{22}^s V_{31}^s + H_{15}^s H_{30}^s H_{42} V_{13} H_{26} V_7 \\
& + H_{29}^s H_{30}^s V_{31}^s V_{32}^s V_{29} V_{30} + H_{30}^s H_{32}^s H_{37}^s H_{39}^s + \Phi_{23} H_{29}^s H_{30}^s V_{24} V_{23} \\
& + \bar{\Phi}_{56} H_{31}^s H_{38}^s H_{42} H_{35} + H_{15}^s H_{30}^s H_{31}^s H_{39} V_{22}^s V_{31}^s + H_{29}^s H_{30}^s V_{21}^s V_{22}^s V_{31}^s V_{32}^s \\
& + H_{30}^s H_{37}^s H_{42} H_{35} + \Phi_{23} H_{29}^s H_{30}^s V_{29} V_{30} + \dots]
\end{aligned}$$

Possible Higgs $h_1 \bar{h}_2$ mass terms (through 8th order):

$$h_1 \bar{h}_2 [\Phi_{12} \quad + \dots]$$

Possible Higgs $h_1 \bar{h}_3$ mass terms (through 8th order):

$$\begin{aligned}
& h_1 \bar{h}_3 [\Phi_{13} \\
& + N_1^c H_{29}^s H_{30}^s H_{30}^s H_{32}^s V_1^s + N_2^c H_{17}^s H_{30}^s H_{30}^s V_4 H_{35} + N_2^c H_{19}^s H_{30}^s H_{32}^s V_{34} H_{35} \\
& + N_3^c H_{16}^s H_{29}^s H_{30}^s H_{26}^s V_{37} + N_3^c H_{29}^s H_{30}^s H_{30}^s H_{32}^s V_{21} + N_3^c H_{29}^s H_{30}^s H_{30}^s V_{24} H_{35} \\
& + H_{17}^s H_{29}^s H_{30}^s H_{30}^s V_4 V_{13} + H_{17}^s H_{29}^s H_{30}^s H_{30}^s V_9 V_{19} + H_{19}^s H_{29}^s H_{30}^s H_{31}^s V_{15} V_{37} \\
& + H_{19}^s H_{29}^s H_{30}^s V_{12}^s V_{34} H_{35} + H_{21}^s H_{29}^s H_{30}^s H_{30}^s V_{19} V_{39} + H_{29}^s H_{30}^s H_{30}^s H_{32}^s H_{37}^s H_{39}^s \\
& + H_{29}^s H_{30}^s H_{30}^s V_{31}^s H_{23} V_{35} + H_{29}^s H_{30}^s H_{37}^s V_{32}^s H_{28} V_{40} + H_{29}^s H_{30}^s H_{37}^s V_{32}^s H_{28} V_{40} \\
& + \dots]
\end{aligned}$$

Possible Higgs $h_1 \bar{h}_4$ mass terms (through 8th order):

$$\begin{aligned}
& h_1 \bar{h}_4 [H_{29}^s H_{30}^s H_{31}^s V_{31}^s V_{32}^s] \\
& + N_1^c \Phi_{12} H_{30}^s H_{31}^s H_{32}^s V_1^s \\
& + N_3^c \Phi_{12} H_{30}^s H_{31}^s V_{24} H_{35} \\
& + \Phi_{12} H_{16}^s H_{29}^s V_{32}^s H_{28} V_{30} \\
& + \Phi_{12} H_{18}^s H_{29}^s H_{30}^s V_1^s V_{12} \\
& + \Phi_{12} H_{19}^s H_{31}^s V_{12}^s V_{34} H_{35} \\
& + \Phi_{12} H_{30}^s H_{31}^s V_{31}^s H_{23} V_{35} \\
& + \dots]
\end{aligned}
\quad
\begin{aligned}
& + N_2^c \Phi_{12} H_{18}^s H_{30}^s H_{32}^s V_1^s \\
& + \Phi_{12} H_{15}^s H_{30}^s H_{31}^s V_{14} V_3 \\
& + \Phi_{12} H_{16}^s H_{29}^s V_{32}^s H_{26} V_{27} \\
& + \Phi_{12} H_{18}^s H_{29}^s H_{30}^s V_7 V_{15} \\
& + \Phi_{12} H_{21}^s H_{30}^s H_{31}^s V_{19} V_{39} \\
& + \Phi_{12} H_{30}^s H_{32}^s V_1^s H_{25} V_9 \\
& + N_3^c \Phi_{12} H_{16}^s H_{31}^s H_{26} V_{37} \\
& + \Phi_{12} H_{16}^s H_{29}^s H_{30}^s V_2^s V_{11} \\
& + \Phi_{12} H_{17}^s H_{30}^s H_{31}^s V_4 V_{13} \\
& + \Phi_{12} H_{19}^s H_{31}^s H_{31}^s V_{15} V_{37} \\
& + \Phi_{12} H_{30}^s H_{31}^s H_{32}^s H_{37}^s H_{39}^s \\
& + \Phi_{12} H_{30}^s V_4 H_{35} H_{23} V_7 \\
& + N_3^c \Phi_{12} H_{30}^s H_{31}^s H_{32}^s V_{21} \\
& + \Phi_{12} H_{16}^s H_{29}^s H_{30}^s V_5 V_{17} \\
& + \Phi_{12} H_{17}^s H_{30}^s H_{31}^s V_9 V_{19} \\
& + \Phi_{12} H_{19}^s H_{31}^s H_{32}^s V_{34} V_{13} \\
& + \Phi_{12} H_{30}^s H_{31}^s H_{37}^s H_{42} H_{35} \\
& + \Phi_{12} H_{31}^s H_{37}^s V_{32}^s H_{28} V_{40}
\end{aligned}$$

Possible Higgs $h_2 \bar{h}_1$ mass terms (through 8th order):

Possible Higgs $h_2 \bar{h}_1$ mass terms continued:

$$\begin{aligned}
& + H_{37}^s V_{32}^s V_{29} V_{40} H_{26} V_{27} & + H_{38}^s V_1^s V_{12}^s V_{22}^s V_{34} H_{35} & + H_{38}^s V_{12}^s V_{34} V_{13} H_{28} V_{39} & + H_{38}^s V_{22}^s V_{34} H_{35} V_7 V_{15} \\
& + H_{38}^s V_4 H_{35} V_{13} V_{23} V_{33} & + H_{38}^s H_{35} V_{13} V_{33} V_5 V_{27} & + H_{38}^s H_{35} V_{23} V_{33} V_9 V_{19} & + H_{38}^s V_{13} V_{13} V_{33} H_{26} V_{35} \\
& + V_1^s V_{12}^s H_{25} V_{30} H_{26} V_{37} & + V_1^s V_{12}^s H_{28} V_{30} H_{23} V_{37} & + V_1^s V_{12}^s H_{23} H_{26} V_{27} V_{37} & + V_{22}^s V_{31}^s V_{34} H_{35} H_{25} V_{30} \\
& + V_{22}^s V_{31}^s V_{34} H_{35} H_{23} V_{27} & + V_4 V_{13} H_{25} H_{28} V_{30} V_{40} & + V_4 V_{13} H_{25} V_{40} H_{26} V_{27} & + V_4 V_{13} H_{28} V_{40} H_{23} V_{27} \\
& + H_{25} H_{28} V_9 V_{19} V_{30} V_{40} & + H_{25} H_{28} V_9 V_{40} V_{15} V_{27} & + H_{25} V_9 V_{19} V_{40} H_{26} V_{27} & + H_{25} V_{30} H_{26} V_7 V_{15} V_{37} \\
& + H_{28} V_9 V_{19} V_{40} H_{23} V_{27} & + H_{28} V_{30} H_{23} V_7 V_{15} V_{37} & + V_{19} V_{30} H_{23} H_{26} V_7 V_{37} & + H_{23} H_{26} V_7 V_{15} V_{27} V_{37} \\
& + \dots]
\end{aligned}$$

Possible Higgs $h_2 \bar{h}_2$ mass terms (through 8th order):

$$\begin{aligned}
& h_2 \bar{h}_2 [N_3^c H_{16}^s H_{28} V_{37} \\
& + N_3^c \bar{\Phi}_4 H_{16}^s H_{28} V_{40} \\
& + \Phi_{13} V_{31}^s V_{32}^s V_{29} V_{30} \\
& + \Phi_4 H_{37}^s V_{32}^s H_{26} V_{37} \\
& + N_1^c H_{16}^s H_{20}^s H_{29}^s H_{30}^s V_{31} \\
& + N_3^c H_{30}^s H_{31}^s V_{31}^s V_{25} V_{37} \\
& + H_{15}^s H_{19}^s H_{22}^s H_{30}^s V_4 V_{13} \\
& + H_{15}^s H_{30}^s H_{31}^s V_{31}^s H_{42} V_{23} \\
& + H_{15}^s H_{30}^s V_{20} V_{39} V_7 V_{37} \\
& + H_{21}^s H_{30}^s V_4 V_3 V_{19} V_{40} \\
& + H_{29}^s H_{30}^s V_{31}^s V_{32}^s V_{24} V_{23} \\
& + H_{30}^s V_{31}^s V_4 V_3 H_{23} V_{37} \\
& + H_{37}^s V_{32}^s H_{28} V_{40} \\
& + \Phi_{13} H_{36}^s H_{37}^s V_{31}^s V_{32}^s \\
& + \Phi_{13} V_{31}^s V_{32}^s V_{25} V_{27} \\
& + N_1^c H_{30}^s H_{32}^s V_{31}^s V_{34} V_3 \\
& + \Phi_{13} \bar{\Phi}_4 H_{16}^s H_{39}^s V_{22}^s V_{31}^s \\
& + H_{15}^s H_{19}^s H_{22}^s H_{30}^s V_9 V_{19} \\
& + H_{15}^s H_{30}^s H_{39}^s V_{12}^s H_{26} V_7 \\
& + H_{15}^s H_{30}^s V_{31}^s V_{32}^s V_{14} V_{33} \\
& + H_{15}^s H_{30}^s V_{20} V_{40} V_7 V_{35} \\
& + H_{21}^s H_{30}^s V_{19} V_{40} V_5 V_7 \\
& + H_{29}^s H_{30}^s H_{36}^s H_{37}^s V_{31}^s V_{32}^s \\
& + H_{29}^s H_{30}^s V_{31}^s V_{32}^s V_{29} V_{30} \\
& + H_{30}^s V_{31}^s H_{23} V_5 V_7 V_{37} \\
& + N_3^c \Phi_{13} \bar{\Phi}_4 H_{16}^s H_{36}^s V_{31}^s \\
& + \Phi_{13} \bar{\Phi}_4 H_{16}^s V_{31}^s H_{42} V_{23} \\
& + H_{15}^s H_{20}^s H_{30}^s V_{31}^s H_{23} V_7 \\
& + H_{15}^s H_{30}^s V_1^s V_{32}^s V_{14} V_{33} \\
& + H_{17}^s H_{30}^s V_2^s V_{31}^s V_{34} V_{13} \\
& + H_{29}^s H_{30}^s H_{36}^s H_{37}^s V_{31}^s V_{32}^s \\
& + H_{29}^s H_{30}^s V_{31}^s V_{32}^s V_{25} V_{27} \\
& + \dots]
\end{aligned}$$

Possible Higgs $h_2 \bar{h}_3$ mass terms (through 8th order):

$$\begin{aligned}
& h_2 \bar{h}_3 [\bar{\Phi}_{23}] \\
& + N_1^c H_{30}^s H_{32}^s V_1^s V_{34} V_{33} \\
& + N_2^c H_{22}^s H_{30}^s H_{37}^s H_{26} V_{37} \\
& + N_2^c H_{30}^s H_{26} V_7 V_{25} V_{37} \\
& + H_{15}^s H_{30}^s V_{14} V_{34} V_3 V_{33} \\
& + H_{17}^s H_{30}^s H_{37}^s V_{25} V_{35} \\
& + H_{17}^s H_{30}^s V_{34} V_{33} V_9 V_{19} \\
& + H_{21}^s H_{30}^s V_{34} V_{33} V_{19} V_{39} \\
& + H_{30}^s H_{37}^s H_{25} V_{39} H_{26} V_{37} \\
& + H_{30}^s V_1^s V_4 V_{33} H_{23} V_{37} \\
& + N_2^c H_{15}^s H_{22}^s H_{30}^s H_{28} V_{30} \\
& + N_2^c H_{30}^s V_2^s V_{21}^s H_{28} V_{40} \\
& + N_3^c H_{16}^s V_{34} V_{33} H_{26} V_{37} \\
& + H_{15}^s H_{30}^s H_{25} H_{28} V_{30} V_{39} \\
& + H_{17}^s H_{30}^s H_{32}^s H_{37}^s V_{34} V_{23} \\
& + H_{19}^s H_{31}^s V_{34} V_{33} V_{15} V_{37} \\
& + H_{29}^s H_{31}^s V_{30} V_{40} V_{27} V_{37} \\
& + H_{30}^s H_{37}^s H_{25} V_{40} H_{26} V_{35} \\
& + H_{30}^s V_{31}^s V_{34} V_{33} H_{23} V_{35} \\
& + N_2^c H_{15}^s H_{22}^s H_{30}^s H_{26} V_{27} \\
& + N_2^c H_{30}^s V_{24} V_3 H_{26} V_{37} \\
& + N_3^c H_{30}^s H_{32}^s V_{21}^s V_{34} V_{33} \\
& + H_{15}^s H_{30}^s H_{25} V_{39} H_{26} V_{27} \\
& + H_{17}^s H_{30}^s H_{37}^s V_{22}^s V_{34} H_{35} \\
& + H_{19}^s H_{32}^s V_{34} V_{33} V_{13} V_{33} \\
& + H_{30}^s H_{32}^s H_{37}^s H_{39}^s V_{34} V_{33} \\
& + H_{30}^s H_{37}^s H_{28} V_{39} H_{23} V_{37} \\
& + H_{37}^s V_{32}^s V_{34} V_{33} H_{28} V_{40} \\
& + \dots
\end{aligned}$$

Possible Higgs $h_2 \bar{h}_4$ mass terms (through 8th order):

$$\begin{aligned}
& h_2 \bar{h}_4 [H_{31}^s \\
& + H_{16}^s H_{19}^s V_2^s V_{34} H_{35} \\
& + N_2^c \Phi_{12} H_{16}^s H_{36}^s H_{28} V_9 \\
& + \Phi_{12} H_{16}^s V_2^s V_{11}^s V_{34} V_{33} \\
& + \Phi_{12} H_{16}^s H_{23} H_{26} V_{27} V_{35} \\
& + \Phi_{12} H_{18}^s V_{34} V_{33} V_7 V_{15} \\
& + \Phi_{12} H_{30}^s V_2^s V_{21}^s V_7 V_{25} \\
& + \Phi_{13} H_{16}^s H_{20}^s H_{21}^s V_1^s V_{12} \\
& + \Phi_{13} H_{16}^s V_{11}^s V_{32}^s V_4 V_{33} \\
& + \Phi_{13} H_{18}^s V_{12}^s V_{31}^s V_{34} V_3 \\
& + \Phi'_4 H_{15}^s H_{18}^s H_{30}^s V_4 V_3 \\
& + \overline{\Phi}_4 H_{16}^s H_{17}^s H_{30}^s V_1^s V_2^s \\
& + \overline{\Phi}'_4 H_{30}^s H_{39}^s V_2^s H_{26} V_7 \\
& + H_{16}^s H_{29}^s V_{32}^s H_{28} V_{30} \\
& + N_3^c \Phi_{13} H_{16}^s H_{18}^s H_{20}^s V_{31} \\
& + \Phi_{12} H_{16}^s V_{34} V_{33} V_5 V_{17} \\
& + \Phi_{12} H_{18}^s H_{30}^s H_{37}^s V_{29} V_{39} \\
& + \Phi_{12} H_{22}^s H_{30}^s H_{37}^s V_2^s V_{21} \\
& + \Phi_{12} H_{30}^s V_{24} V_3 V_9 V_{29} \\
& + \Phi_{13} H_{16}^s H_{20}^s H_{21}^s V_7 V_{15} \\
& + \Phi_{13} H_{16}^s H_{42} V_{13} H_{28} V_9 \\
& + \Phi_{13} V_{12}^s V_{21}^s V_{32}^s H_{26} V_7 \\
& + \Phi'_4 H_{15}^s H_{18}^s H_{30}^s V_5 V_7 \\
& + \overline{\Phi}_4 H_{16}^s H_{17}^s H_{30}^s V_4 V_3 \\
& + \overline{\Phi}'_4 H_{30}^s H_{42} V_3 H_{28} V_9 \\
& + H_{16}^s H_{29}^s V_{32}^s H_{26} V_{27} \\
& + N_3^c \overline{\Phi}'_{56} H_{22}^s H_{30}^s H_{28} V_9 \\
& + \Phi_{12} H_{16}^s H_{28} V_{30} H_{23} V_{35} \\
& + \Phi_{12} H_{18}^s V_1^s V_{12}^s V_{34} V_{33} \\
& + \Phi_{12} H_{30}^s V_2^s V_{21}^s V_{24} V_3 \\
& + \Phi_{13} H_{16}^s H_{18}^s H_{19}^s V_{15} V_{35} \\
& + \Phi_{13} H_{16}^s H_{39}^s V_{12}^s H_{28} V_9 \\
& + \Phi_{13} H_{16}^s V_9 V_{39} V_{17} V_{37} \\
& + \Phi_{13} V_{32}^s V_{24} V_{13} H_{26} V_7 \\
& + \Phi'_4 H_{15}^s H_{18}^s H_{30}^s V_1^s V_2^s \\
& + \overline{\Phi}_4 H_{16}^s H_{16}^s H_{19}^s H_{20}^s H_{29}^s \\
& + \overline{\Phi}'_4 H_{16}^s H_{19}^s H_{32}^s V_{34} V_3 \\
& + \dots]
\end{aligned}$$

Possible Higgs $h_3 \bar{h}_1$ mass terms (through 8th order):

$$\begin{aligned}
& h_3 \bar{h}_1 [\bar{\Phi}_1 \\
& + N_1^c H_{30}^s H_{32}^s V_1^s V_{11}^s V_{12}^s \\
& + N_1^c H_{30}^s H_{32}^s V_{11}^s V_7 V_{15} \\
& + N_3^c H_{16}^s V_{14} V_{13} H_{26} V_{37} \\
& + N_3^c H_{30}^s H_{32}^s V_{11}^s V_{12}^s V_{21} \\
& + N_3^c H_{30}^s H_{32}^s V_{21}^s V_{15} V_{17} \\
& + N_3^c H_{30}^s V_{24} H_{35} V_{19} V_{20} \\
& + H_{15}^s H_{30}^s V_{14} V_3 V_{19} V_{20} \\
& + H_{17}^s H_{30}^s V_4 V_{14} V_{13} V_{13} \\
& + H_{17}^s H_{30}^s V_9 V_{19} V_{19} V_{20} \\
& + H_{19}^s H_{31}^s V_{19} V_{20} V_{15} V_{37} \\
& + H_{19}^s H_{32}^s V_{21} V_{32} V_{15} V_{27} \\
& + H_{19}^s V_{11}^s V_{12}^s V_{12}^s V_{34} H_{35} \\
& + H_{19}^s V_{32}^s V_{24} H_{35} V_{19} V_{30} \\
& + H_{21}^s H_{30}^s V_{19} V_{19} V_{20} V_{39} \\
& + H_{21}^s V_{22}^s V_{31}^s H_{35} V_{13} V_{23} \\
& + H_{30}^s H_{32}^s H_{37}^s H_{39}^s V_{15} V_{17} \\
& + H_{30}^s H_{37}^s H_{42}^s V_{14} H_{35} V_{13} \\
& + H_{30}^s H_{39}^s V_2^s V_{31}^s V_{24} V_{13} \\
& + H_{30}^s V_{31}^s H_{42}^s V_{13} V_9 V_{29} \\
& + H_{30}^s V_{31}^s H_{23}^s V_{15} V_{17} V_{35} \\
& + H_{37}^s V_{32}^s V_{14} V_{13} H_{28} V_{40} \\
& + H_{38}^s V_2^s V_{31}^s H_{35} V_{13} V_{23} \\
& + N_1^c H_{30}^s H_{32}^s V_1^s V_{14} V_{13} \\
& + N_1^c H_{30}^s V_1^s V_{12}^s V_{14} H_{35} \\
& + N_3^c H_{16}^s H_{28} V_{20} V_{15} V_{37} \\
& + N_3^c H_{30}^s H_{32}^s V_{11}^s V_{24} V_{13} \\
& + N_3^c H_{30}^s V_{11}^s V_{12}^s V_{24} H_{35} \\
& + N_3^c H_{30}^s V_{24} H_{35} V_{15} V_{17} \\
& + H_{15}^s H_{30}^s V_{14} V_3 V_{15} V_{17} \\
& + H_{17}^s H_{30}^s V_4 V_{13} V_{19} V_{20} \\
& + H_{17}^s H_{30}^s V_9 V_{19} V_{15} V_{17} \\
& + H_{19}^s H_{31}^s V_{15} V_{15} V_{17} V_{37} \\
& + H_{19}^s H_{32}^s V_{14} V_{34} V_{13} V_{13} \\
& + H_{19}^s V_{12}^s V_{14} V_{34} H_{35} V_{13} \\
& + H_{19}^s V_{32}^s V_{24} H_{35} V_{15} V_{27} \\
& + H_{21}^s H_{30}^s V_{19} V_{39} V_{15} V_{17} \\
& + H_{30}^s H_{32}^s H_{37}^s H_{39}^s V_{11}^s V_{12}^s \\
& + H_{30}^s H_{32}^s H_{37}^s V_{11}^s H_{42}^s V_{13} \\
& + H_{30}^s H_{37}^s H_{42}^s H_{35} V_{19} V_{20} \\
& + H_{30}^s H_{39}^s V_{12}^s V_{31}^s V_9 V_{29} \\
& + H_{30}^s V_{31}^s V_{14} V_{13} H_{23} V_{35} \\
& + H_{32}^s H_{38}^s V_1^s V_{32}^s V_{19} V_{30} \\
& + H_{37}^s V_{32}^s H_{28} V_{19} V_{20} V_{40} \\
& + H_{37}^s V_{32}^s V_{11}^s V_{12}^s V_{32}^s H_{28} V_{40} \\
& + H_{37}^s V_{32}^s V_{19} V_{40} H_{26} V_{17} \\
& + N_1^c H_{30}^s H_{32}^s V_1^s V_{15} V_{17} \\
& + N_3^c H_{16}^s V_{11}^s V_{12}^s H_{26} V_{37} \\
& + N_3^c H_{16}^s H_{26} V_{15} V_{17} V_{37} \\
& + N_3^c H_{30}^s H_{32}^s V_{21}^s V_{19} V_{20} \\
& + N_3^c H_{30}^s V_{14} V_{24} H_{35} V_{13} \\
& + H_{15}^s H_{30}^s V_{14} V_{14} V_{13} \\
& + H_{17}^s H_{30}^s V_{11}^s V_{12}^s V_9 V_{19} \\
& + H_{17}^s H_{30}^s V_{14} V_{13} V_9 V_{19} \\
& + H_{19}^s H_{31}^s V_{11}^s V_{12}^s V_{15} V_{37} \\
& + H_{19}^s H_{32}^s V_{11}^s V_{12}^s V_{34} V_{13} \\
& + H_{19}^s H_{32}^s V_{34} V_{13} V_{19} V_{20} \\
& + H_{19}^s V_{12}^s V_{34} H_{35} V_{19} V_{20} \\
& + H_{21}^s H_{30}^s V_{11}^s V_{12}^s V_{19} V_{39} \\
& + H_{21}^s H_{32}^s V_{31}^s V_{13} V_{23} V_{23} \\
& + H_{30}^s H_{32}^s H_{37}^s H_{39}^s V_{14} V_{13} \\
& + H_{30}^s H_{37}^s H_{39}^s V_{12}^s V_{14} H_{35} \\
& + H_{30}^s H_{37}^s H_{42}^s H_{35} V_{15} V_{17} \\
& + H_{30}^s V_{21}^s V_{31}^s H_{42} V_{13} \\
& + H_{30}^s V_{31}^s H_{25} V_{20} V_{15} V_{35} \\
& + H_{32}^s H_{38}^s V_1^s V_{32}^s V_{15} V_{27} \\
& + H_{37}^s V_{32}^s H_{28} V_{40} V_{15} V_{17} \\
& + \dots]
\end{aligned}$$

Possible Higgs $h_3 \bar{h}_2$ mass terms (through 8th order):

$$\begin{aligned}
& h_3 \bar{h}_2 [\Phi_{23} \\
& + N_1^c H_{30}^s H_{32}^s V_1^s V_{31}^s V_{32}^s \\
& + H_{15}^s H_{30}^s V_{31}^s V_{32}^s V_{14} V_3 \\
& + H_{19}^s H_{32}^s V_{31}^s V_{32}^s V_{34} V_{13} \\
& + H_{30}^s H_{37}^s V_{31}^s V_{32}^s H_{42} H_{35} \\
& + N_3^c H_{16}^s V_{31}^s V_{32}^s H_{26} V_{37} \\
& + H_{17}^s H_{30}^s V_{31}^s V_{32}^s V_4 V_{13} \\
& + H_{19}^s V_{12}^s V_{31}^s V_{32}^s V_{34} H_{35} \\
& + H_{30}^s V_{31}^s V_{32}^s H_{23} V_{35} \\
& + N_3^c H_{30}^s H_{32}^s V_{21}^s V_{31}^s V_{32}^s \\
& + H_{17}^s H_{30}^s V_{31}^s V_{32}^s V_9 V_{19} \\
& + H_{21}^s H_{30}^s V_{31}^s V_{32}^s V_{19} V_{39} \\
& + H_{37}^s V_{31}^s V_{32}^s V_{32}^s H_{28} V_{40} \\
& + N_3^c H_{30}^s V_{31}^s V_{32}^s V_{24} H_{35} \\
& + H_{19}^s H_{31}^s V_{31}^s V_{32}^s V_{15} V_{37} \\
& + H_{30}^s H_{32}^s H_{37}^s H_{39}^s V_{31}^s V_{32}^s \\
& + \dots]
\end{aligned}$$

Possible Higgs $h_3 \bar{h}_3$ mass terms (through 8th order):

$$\begin{aligned}
& h_3 \bar{h}_3 [N_1^c H_{30}^s H_{32}^s V_1^s \\
& + H_{19}^s H_{31}^s V_{15} V_{37} \\
& + H_{30}^s V_{31}^s H_{23} V_{35} \\
& + N_1^c \Phi_4' H_{30}^s V_4 H_{35} \\
& + \Phi_{12} V_{34} V_{33} V_{19} V_{20} \\
& + \Phi_4' H_{15}^s H_{30}^s V_5 V_{17} \\
& + \bar{\Phi}_4' H_{19}^s H_{31}^s V_{19} V_{40} \\
& + N_1^c H_{16}^s H_{20}^s H_{29}^s H_{30}^s V_{31}^s \\
& + \Phi_{12} \Phi_4' V_{12}^s V_{14} V_{34} V_{34} \\
& + H_{15}^s H_{19}^s H_{22}^s H_{30}^s V_9 V_{19} \\
& + H_{15}^s H_{30}^s H_{39}^s V_{12}^s H_{26} V_7 \\
& + H_{15}^s H_{30}^s V_{20}^s V_{40} V_7 V_{35} \\
& + H_{21}^s H_{30}^s V_{19} V_{40} V_5 V_7 \\
& + H_{29}^s H_{30}^s V_{31}^s V_{32}^s V_{29} V_{30} \\
& + H_{30}^s V_{31}^s H_{23} V_5 V_7 V_{37} \\
& + H_{15}^s H_{30}^s V_{14} V_3 \\
& + H_{19}^s H_{32}^s V_{34} V_{13} \\
& + N_2^c \bar{\Phi}_4' H_{22}^s H_{30}^s V_{31}^s \\
& + \Phi_{12} V_{34} V_{33} V_{15} V_{17} \\
& + \Phi_4' H_{21}^s H_{30}^s V_{15} V_{35} \\
& + \bar{\Phi}_4' H_{19}^s H_{35} V_{13} V_{33} \\
& + N_1^c H_{30}^s H_{32}^s V_{31}^s V_{34} V_3 \\
& + \Phi_{12} \bar{\Phi}_4' V_{11}^s V_{13} V_{33} V_{33} \\
& + H_{15}^s H_{20}^s H_{30}^s V_{31}^s H_{23} V_7 \\
& + H_{15}^s H_{30}^s V_1^s V_{32}^s V_{14} V_{33} \\
& + H_{17}^s H_{30}^s V_2^s V_{31}^s V_{34} V_{13} \\
& + H_{29}^s H_{30}^s H_{36}^s H_{37}^s V_{31}^s V_{32}^s \\
& + H_{29}^s H_{30}^s V_{31}^s V_{32}^s V_{25} V_{27} \\
& + \dots]
\end{aligned}
\begin{aligned}
& + H_{17}^s H_{30}^s V_4 V_{13} \\
& + H_{19}^s V_{12}^s V_{34} H_{35} \\
& + \Phi_{12} V_{11}^s V_{12}^s V_{34} V_{33} \\
& + \bar{\Phi}_{12} H_{29}^s H_{30}^s V_{31}^s V_{32}^s \\
& + \bar{\Phi}_4' H_{17}^s H_{30}^s V_1^s V_{12}^s \\
& + \bar{\Phi}_4' H_{30}^s V_{31}^s H_{25} V_{39} \\
& + N_3^c H_{15}^s H_{30}^s H_{31}^s H_{36}^s V_{31}^s \\
& + H_{15}^s H_{18}^s H_{19}^s H_{30}^s V_{19} V_{39} \\
& + H_{15}^s H_{30}^s H_{31}^s H_{39}^s V_{22} V_{31}^s \\
& + H_{15}^s H_{30}^s H_{42}^s V_{13} H_{26} V_7 \\
& + H_{21}^s H_{30}^s V_1^s V_2^s V_{19} V_{40} \\
& + H_{29}^s H_{30}^s V_{21}^s V_{22} V_{31}^s V_{32}^s \\
& + H_{30}^s V_1^s V_2^s V_{31}^s H_{23} V_{37} \\
& + N_3^c H_{30}^s H_{31}^s V_{31}^s V_{25} V_{37} \\
& + H_{15}^s H_{19}^s H_{22}^s H_{30}^s V_4 V_{13} \\
& + H_{15}^s H_{30}^s H_{31}^s V_{31}^s H_{42} V_{23} \\
& + H_{15}^s H_{30}^s V_{20} V_{39} V_7 V_{37} \\
& + H_{21}^s H_{30}^s V_4 V_3 V_{19} V_{40} \\
& + H_{29}^s H_{30}^s V_{31}^s V_{32}^s V_{24} V_{23} \\
& + H_{30}^s V_{31}^s V_4 V_3 H_{23} V_{37}
\end{aligned}$$

Possible Higgs $h_3 \bar{h}_4$ mass terms (through 8th order):

$$\begin{aligned}
& h_3 \bar{h}_3 [H_{31}^s V_{31}^s V_{32}^s \\
& + \Phi_{12} H_{16}^s V_2^s V_{11}^s \\
& + \Phi_{12} \Phi'_4 H_{18}^s V_4 V_{13} \\
& + N_1^c \Phi_{12} H_{16}^s H_{20}^s H_{31}^s V_{31}^s \\
& + N_3^c \Phi_{23} H_{16}^s H_{31}^s H_{26}^s V_{37} \\
& + \Phi_{12} H_{16}^s H_{20}^s H_{21}^s V_1^s V_{12}^s \\
& + \Phi_{12} H_{16}^s V_{11}^s V_{32}^s V_4 V_{33} \\
& + \Phi_{12} H_{18}^s V_{12}^s V_{31}^s V_{34} V_3 \\
& + \Phi_{13} H_{16}^s V_{31}^s V_{32}^s V_5 V_{17} \\
& + \Phi_{23} H_{16}^s H_{29}^s H_{30}^s V_2^s V_{11}^s \\
& + \Phi_{23} H_{17}^s H_{30}^s H_{31}^s V_4 V_{13} \\
& + \Phi_{23} H_{19}^s H_{31}^s H_{31}^s V_{15} V_{37} \\
& + \Phi_{23} H_{30}^s H_{31}^s H_{32}^s H_{37}^s H_{39}^s \\
& + \Phi_{23} H_{30}^s V_4 H_{35}^s H_{23} V_7 \\
& + \Phi_{12} H_{16}^s V_5 V_{17} \\
& + \Phi_{12} \Phi'_4 H_{18}^s V_9 V_{19} \\
& + N_1^c \Phi_{23} H_{30}^s H_{31}^s H_{32}^s V_1^s \\
& + N_3^c \Phi_{23} H_{30}^s H_{31}^s H_{32}^s V_{21}^s \\
& + \Phi_{12} H_{16}^s H_{20}^s H_{21}^s V_7 V_{15} \\
& + \Phi_{12} H_{16}^s H_{42} V_{13} H_{28} V_9 \\
& + \Phi_{12} V_{12}^s V_{21}^s V_{32}^s H_{26} V_7 \\
& + \Phi_{13} H_{18}^s V_1^s V_{12}^s V_{31}^s V_{32}^s \\
& + \Phi_{23} H_{16}^s H_{29}^s H_{30}^s V_5 V_{17} \\
& + \Phi_{23} H_{17}^s H_{30}^s H_{31}^s V_9 V_{19} \\
& + \Phi_{23} H_{19}^s H_{31}^s H_{32}^s V_{34} V_{13} \\
& + \Phi_{23} H_{30}^s H_{31}^s H_{37}^s H_{42} H_{35} \\
& + \Phi_{23} H_{31}^s H_{37}^s V_{32}^s H_{28} V_{40} \\
& + \Phi_{12} H_{18}^s V_1^s V_{12}^s \\
& + \Phi_{12} \Phi'_4 H_{16}^s V_{14} V_3 \\
& + N_2^c \Phi_{12} H_{16}^s H_{18}^s H_{20}^s V_{31}^s \\
& + N_3^c \Phi_{23} H_{30}^s H_{31}^s V_{24} H_{35} \\
& + \Phi_{12} H_{16}^s H_{20}^s V_{31}^s H_{25} V_9 \\
& + \Phi_{12} H_{16}^s V_9 V_{39} V_{17} V_{37} \\
& + \Phi_{12} V_{32}^s V_{24} V_{13} H_{26} V_7 \\
& + \Phi_{13} H_{18}^s V_{31}^s V_{32}^s V_7 V_{15} \\
& + \Phi_{23} H_{16}^s H_{29}^s V_{32}^s H_{28} V_{30} \\
& + \Phi_{23} H_{18}^s H_{29}^s H_{30}^s V_1^s V_{12}^s \\
& + \Phi_{23} H_{19}^s H_{31}^s V_{12}^s V_{34} H_{35} \\
& + \Phi_{23} H_{30}^s H_{31}^s V_{31}^s H_{23} V_{35} \\
& + \Phi_{12} H_{18}^s V_7 V_{15} \\
& + H_{16}^s H_{19}^s V_{32}^s V_4 H_{35} \\
& + N_2^c \Phi_{23} H_{18}^s H_{30}^s H_{32}^s V_1^s \\
& + \Phi_{12} H_{16}^s H_{18}^s H_{19}^s V_{15} V_{35} \\
& + \Phi_{12} H_{16}^s H_{39}^s V_{12}^s H_{28} V_9 \\
& + \Phi_{12} H_{16}^s V_9 V_{40} V_{17} V_{35} \\
& + \Phi_{13} H_{16}^s V_2^s V_{11}^s V_{31}^s V_{32}^s \\
& + \Phi_{23} H_{15}^s H_{30}^s H_{31}^s V_{14} V_3 \\
& + \Phi_{23} H_{16}^s H_{29}^s V_{32}^s H_{26} V_{27} \\
& + \Phi_{23} H_{18}^s H_{29}^s H_{30}^s V_7 V_{15} \\
& + \Phi_{23} H_{21}^s H_{30}^s H_{31}^s V_{19} V_{39} \\
& + \Phi_{23} H_{30}^s H_{32}^s V_1^s H_{25} V_9 \\
& + \dots]
\end{aligned}$$

Possible Higgs $h_4 \bar{h}_1$ mass terms (through 8th order):

$$\begin{aligned}
& h_4 \bar{h}_1 [H_{38}^s V_{11}^s V_{12}^s \\
& + N_1^c H_{30}^s V_{31}^s V_{20} V_{29} \\
& + H_{36}^s H_{37}^s H_{38}^s V_{31}^s V_{32}^s \\
& + H_{38}^s V_{31}^s V_{32}^s V_{29} V_{30} \\
& + N_1^c \bar{\Phi}_{13} H_{19}^s H_{30}^s H_{32}^s V_{21} \\
& + N_1^c \bar{\Phi}_{23} H_{30}^s V_{21}^s V_{14} V_{33} \\
& + N_3^c \bar{\Phi}_{13} H_{16}^s H_{38}^s H_{26} V_{37} \\
& + \bar{\Phi}_{12} H_{21}^s H_{30}^s H_{39}^s V_{31}^s V_{31}^s \\
& + \bar{\Phi}_{13} H_{15}^s H_{30}^s H_{38}^s V_{14} V_3 \\
& + \bar{\Phi}_{13} H_{17}^s H_{30}^s H_{38}^s V_9 V_{19} \\
& + \bar{\Phi}_{13} H_{21}^s H_{30}^s H_{38}^s V_{19} V_{39} \\
& + \bar{\Phi}_{13} H_{37}^s H_{38}^s V_{32}^s H_{28} V_{40} \\
& + \bar{\Phi}_4 H_{16}^s H_{38}^s V_{31}^s H_{42} V_{23} \\
& + \bar{\Phi}_{56} H_{21}^s H_{36}^s V_{31}^s H_{23} V_{17} \\
& + \bar{\Phi}_{56} H_{38}^s V_{14} H_{35} H_{23} V_{17} \\
& + H_{38}^s V_{14} V_{13} \\
& + N_1^c H_{30}^s V_{31}^s V_{17} V_{25} \\
& + H_{37}^s V_{20} V_{40} V_{17} V_{37} \\
& + H_{38}^s V_{31}^s V_{32}^s V_{25} V_{27} \\
& + N_1^c \bar{\Phi}_{13} H_{19}^s H_{30}^s V_{24} H_{35} \\
& + N_1^c \bar{\Phi}_{56} H_{30}^s V_{31}^s V_{15} V_{27} \\
& + N_3^c \bar{\Phi}_{13} H_{30}^s H_{32}^s H_{38}^s V_{21} \\
& + \bar{\Phi}_{12} H_{21}^s H_{30}^s V_{31}^s H_{42} V_{13} \\
& + \bar{\Phi}_{13} H_{17}^s H_{21}^s H_{30}^s V_{19} V_{30} \\
& + \bar{\Phi}_{13} H_{19}^s H_{31}^s H_{38}^s V_{15} V_{37} \\
& + \bar{\Phi}_{13} H_{30}^s H_{32}^s H_{37}^s H_{38}^s H_{39} \\
& + \Phi_4 H_{15}^s V_{20} V_{20} V_{27} V_{37} \\
& + \bar{\Phi}_4 H_{37}^s V_{40} V_{40} V_{17} V_{17} \\
& + \bar{\Phi}_{56} H_{32}^s H_{38}^s V_{11}^s H_{25} V_{20} \\
& + \dots] \\
& + H_{38}^s V_{19} V_{20} \\
& + H_{15}^s V_{20} V_{40} V_{17} V_{27} \\
& + H_{38}^s V_{21}^s V_{22}^s V_{31}^s V_{32}^s \\
& + H_{38}^s V_{15} V_{17} \\
& + H_{15}^s V_{30} V_{40} V_{17} V_{17} \\
& + H_{38}^s V_{31}^s V_{32}^s V_{24} V_{23} \\
& + N_1^c \Phi_{23} H_{30}^s V_{11}^s V_{24} V_{33} \\
& + N_2^c \bar{\Phi}_{13} H_{21}^s H_{30}^s H_{36}^s V_{31}^s \\
& + N_3^c \bar{\Phi}_4 H_{16}^s H_{36}^s H_{38}^s V_{31}^s \\
& + \bar{\Phi}_{13} H_{15}^s H_{19}^s H_{30}^s V_{17} V_{25} \\
& + \bar{\Phi}_{13} H_{17}^s H_{30}^s H_{38}^s V_4 V_{13} \\
& + \bar{\Phi}_{13} H_{19}^s H_{38}^s V_{12} V_{34} H_{35} \\
& + \bar{\Phi}_{13} H_{30}^s H_{38}^s V_{31}^s H_{23} V_{35} \\
& + \bar{\Phi}_4 H_{16}^s H_{38}^s H_{39}^s V_{22}^s V_{31}^s \\
& + \bar{\Phi}_{56} H_{21}^s H_{36}^s V_{31}^s H_{25} V_{20} \\
& + \bar{\Phi}_{56} H_{38}^s V_{14} H_{35} H_{25} V_{20}
\end{aligned}$$

Possible Higgs $h_4 \bar{h}_2$ mass terms (through 8th order):

$$\begin{aligned}
& h_4 \bar{h}_2 [H_{38}^s V_{31}^s V_{32}^s \\
& + H_{21}^s H_{30}^s H_{39}^s V_{12}^s V_{31}^s \\
& + N_1^c \Phi_{12} H_{16}^s H_{20}^s H_{38}^s V_{31}^s \\
& + N_1^c \Phi_{23} H_{19}^s H_{30}^s V_{24}^s H_{35} \\
& + N_3^c \Phi_{23} H_{30}^s H_{32}^s H_{38}^s V_{21}^s \\
& + \Phi_{12} H_{37}^s V_{20}^s V_{40}^s V_{17}^s V_{37} \\
& + \Phi_{23} H_{17}^s H_{21}^s H_{30}^s V_{19}^s V_{30} \\
& + \Phi_{23} H_{19}^s H_{31}^s H_{38}^s V_{15}^s V_{37} \\
& + \Phi_{23} H_{30}^s H_{32}^s H_{37}^s H_{38}^s H_{39}^s \\
& + \dots]
\end{aligned}
\quad
\begin{aligned}
& + H_{21}^s H_{30}^s V_{31}^s H_{42}^s V_{13} \\
& + N_1^c \Phi_{12} H_{30}^s V_{31}^s V_{20}^s V_{29} \\
& + N_1^c \Phi_{23} H_{30}^s H_{32}^s H_{38}^s V_1^s \\
& + N_3^c \Phi_{23} H_{30}^s H_{38}^s V_{24}^s H_{35} \\
& + \Phi_{12} H_{15}^s V_{20}^s V_{40}^s V_{17}^s V_{27} \\
& + \Phi_{23} H_{15}^s H_{19}^s H_{30}^s V_{20}^s V_{29} \\
& + \Phi_{23} H_{17}^s H_{21}^s H_{30}^s V_{15}^s V_{27} \\
& + \Phi_{23} H_{19}^s H_{32}^s H_{38}^s V_{34}^s V_{13} \\
& + \Phi_{23} H_{30}^s H_{37}^s H_{38}^s H_{42}^s H_{35} \\
& + N_1^c \Phi_{12} H_{30}^s V_{31}^s V_{17}^s V_{25} \\
& + N_2^c \Phi_{23} H_{21}^s H_{30}^s H_{36}^s V_{31}^s \\
& + \Phi_{12} H_{15}^s V_{30}^s V_{40}^s V_{17}^s V_{17} \\
& + \Phi_{23} H_{15}^s H_{30}^s H_{38}^s V_{14}^s V_{3} \\
& + \Phi_{23} H_{17}^s H_{30}^s H_{38}^s V_{9}^s V_{19} \\
& + \Phi_{23} H_{21}^s H_{30}^s H_{38}^s V_{19}^s V_{39} \\
& + \Phi_{23} H_{37}^s H_{38}^s V_{32}^s H_{28}^s V_{40}
\end{aligned}$$

Possible Higgs $h_4 \bar{h}_3$ mass terms (through 8th order):

$$\begin{aligned}
& h_4 \bar{h}_3 [H_{38}^s \\
& + N_1^c H_{16}^s H_{19}^s H_{26} V_{37} \\
& + H_{15}^s H_{19}^s H_{30}^s V_{20} V_{29} \\
& + H_{21}^s V_{30} V_{40} V_7 V_{37} \\
& + N_1^c \Phi_{12} H_{30}^s V_{11}^s V_{24} V_{33} \\
& + N_1^c \Phi_4 H_{15}^s H_{30}^s H_{28} V_{20} \\
& + \Phi_{13} H_{15}^s V_{20} V_{40} V_{17} V_{27} \\
& + \bar{\Phi}_{23} H_{21}^s H_{30}^s V_{31} H_{42} V_{13} \\
& + \bar{\Phi}_4 H_{16}^s H_{19}^s H_{19}^s V_{25} V_{37} \\
& + \Phi_4' H_{15}^s H_{21}^s H_{30}^s V_{14} V_{23} \\
& + \bar{\Phi}_4 H_{17}^s H_{19}^s H_{30}^s V_{24} V_{13} \\
& + \bar{\Phi}_4 H_{17}^s H_{19}^s H_{30}^s V_{12} V_{21} \\
& + \Phi_{56} H_{15}^s H_{19}^s H_{30}^s V_{15} V_{27} \\
& + \bar{\Phi}_{56} H_{21}^s H_{25} H_{28} V_{20} V_{40} \\
& + N_1^c H_{19}^s H_{30}^s H_{32}^s V_{21}^s \\
& + H_{15}^s H_{19}^s H_{30}^s V_{17} V_{25} \\
& + N_1^c H_{19}^s H_{30}^s V_{24} H_{35} \\
& + H_{17}^s H_{21}^s H_{30}^s V_{19} V_{30} \\
& + N_2^c H_{21}^s H_{30}^s H_{36}^s V_{31}^s \\
& + H_{17}^s H_{21}^s H_{30}^s V_{15} V_{27} \\
& + N_1^c \Phi_{13} H_{30}^s V_{31}^s V_{20} V_{29} \\
& + N_1^c \bar{\Phi}_4 H_{15}^s H_{30}^s H_{26} V_{17} \\
& + \Phi_{13} H_{37}^s V_{20} V_{40} V_{17} V_{37} \\
& + \bar{\Phi}_4 H_{21}^s V_9 V_{40} V_{27} V_{37} \\
& + \bar{\Phi}_4 H_{17}^s H_{30}^s V_{31}^s H_{23} V_{27} \\
& + \bar{\Phi}_4 H_{17}^s H_{30}^s V_{31}^s H_{25} V_{30} \\
& + \bar{\Phi}_{56} H_{17}^s H_{21}^s H_{30}^s V_{20} V_{29} \\
& + \bar{\Phi}_{56} H_{21}^s H_{28} V_{40} H_{23} V_{17} \\
& + N_1^c \bar{\Phi}_{13} H_{30}^s V_{31}^s V_{17} V_{25} \\
& + N_2^c \bar{\Phi}_{13} H_{21}^s H_{30}^s H_{28} V_{40} \\
& + \bar{\Phi}_{23} H_{21}^s H_{30}^s H_{39}^s V_{12}^s V_{31}^s \\
& + \bar{\Phi}_4 H_{15}^s H_{16}^s H_{19}^s H_{19}^s H_{36}^s \\
& + \bar{\Phi}_4 H_{21}^s V_{40} V_{40} V_7 V_{27} \\
& + \Phi_{56} H_{15}^s H_{19}^s H_{30}^s V_{19} V_{30} \\
& + \bar{\Phi}_{56} H_{17}^s H_{21}^s H_{30}^s V_{17} V_{25} \\
& + \bar{\Phi}_{56} H_{19}^s H_{30}^s H_{42} V_{14} V_{34} \\
& + \dots]
\end{aligned}$$

Possible Higgs $h_4 \bar{h}_4$ mass terms (through 8th order):

$$\begin{aligned}
& h_4 \bar{h}_4 [p23 H_{31}^s H_{38}^s \\
& + \Phi_{23} \Phi_{56} H_{30}^s H_{37}^s \\
& + N_1^c \Phi_{12} H_{16}^s H_{28} V_{20} \\
& + \Phi_{12} H_{16}^s H_{38}^s V_2^s V_{11}^s \\
& + \Phi_{12} H_{18}^s H_{38}^s V_1^s V_{12}^s \\
& + \Phi_{12} H_{31}^s H_{38}^s V_{11}^s V_{12}^s \\
& + \Phi_{23} H_{17}^s H_{18}^s H_{30}^s H_{37}^s \\
& + \Phi_{56} H_{30}^s H_{37}^s V_{31}^s V_{32}^s \\
& + N_1^c \Phi_{13} \Phi_{23} H_{16}^s H_{28} V_{20} \\
& + N_3^c H_{16}^s H_{21}^s H_{30}^s V_{14} H_{35} \\
& + \Phi_{12} \bar{\Phi}_{13} H_{21}^s H_{30}^s V_7 V_{25} \\
& + \Phi_{12} \Phi_4' H_{18}^s H_{38}^s V_4 V_{13} \\
& + \Phi_{12} \Phi_{56} H_{30}^s H_{37}^s V_{11}^s V_{12}^s \\
& + \Phi_{12} \Phi_5' H_{16}^s H_{21}^s V_{12}^s V_{21} \\
& + \Phi_{13} \Phi_{23} H_{16}^s H_{21}^s V_{11}^s V_{22}^s \\
& + \Phi_{13} \Phi_{23} H_{18}^s H_{19}^s V_{12}^s V_{21} \\
& + \Phi_{13} \Phi_{23} H_{18}^s V_{31}^s H_{25} V_{30} \\
& + H_{15}^s H_{16}^s H_{19}^s V_{32}^s H_{28} V_{20} \\
& + H_{17}^s H_{18}^s H_{30}^s H_{37}^s V_{31}^s V_{32}^s \\
& + H_{21}^s H_{30}^s V_{31}^s V_{32}^s V_{24} V_3 \\
& + H_{31}^s H_{38}^s V_{31}^s V_{32}^s \\
& + N_1^c \Phi_{12} H_{16}^s H_{26} V_{17} \\
& + \Phi_{12} H_{16}^s H_{38}^s V_5 V_{17} \\
& + \Phi_{12} H_{18}^s H_{38}^s V_7 V_{15} \\
& + \Phi_{12} H_{31}^s H_{38}^s V_{14} V_{13} \\
& + \Phi_{23} H_{21}^s H_{22}^s H_{30}^s H_{37}^s \\
& + \bar{\Phi}_{56} H_{19}^s V_{32}^s H_{28} V_{20} \\
& + N_1^c \Phi_{13} \Phi_{23} H_{16}^s H_{26} V_{17} \\
& + \Phi_{12} \bar{\Phi}_{13} H_{17}^s H_{18}^s H_{30}^s H_{37}^s \\
& + \Phi_{12} \Phi_4 H_{18}^s H_{21}^s V_{15} V_{27} \\
& + \Phi_{12} \Phi_4' H_{18}^s H_{38}^s V_9 V_{19} \\
& + \Phi_{12} \Phi_{56} H_{30}^s H_{37}^s V_{14} V_{13} \\
& + \Phi_{12} \Phi_5' H_{16}^s H_{21}^s V_{24} V_{13} \\
& + \Phi_{13} \Phi_{23} H_{16}^s H_{21}^s V_{14} V_{23} \\
& + \Phi_{13} \Phi_{23} H_{18}^s H_{19}^s V_{24} V_{13} \\
& + \Phi_{13} \Phi_{23} H_{18}^s V_{31}^s H_{23} V_{27} \\
& + H_{15}^s H_{16}^s H_{19}^s V_{32}^s H_{26} V_{17} \\
& + H_{21}^s H_{22}^s H_{30}^s H_{37}^s V_{31}^s V_{32}^s \\
& + H_{21}^s H_{30}^s H_{31}^s H_{39}^s V_{12}^s V_{31}^s \\
& + H_{21}^s H_{30}^s V_{31}^s V_{32}^s V_7 V_{25} \\
& + \Phi_{12} H_{16}^s H_{21}^s V_{11}^s V_{22}^s \\
& + \Phi_{12} H_{18}^s H_{19}^s V_{12}^s V_{21}^s \\
& + \Phi_{12} H_{18}^s V_{31}^s H_{25} V_{30} \\
& + \Phi_{12} H_{31}^s H_{38}^s V_{19} V_{20} \\
& + \Phi_{23} H_{21}^s H_{30}^s V_{24} V_3 \\
& + \bar{\Phi}_{56} H_{19}^s V_{32}^s H_{26} V_{17} \\
& + N_3^c \bar{\Phi}_4 \Phi_{56} H_{16}^s H_{30}^s V_{31}^s \\
& + \Phi_{12} \bar{\Phi}_{13} H_{21}^s H_{22}^s H_{30}^s H_{37}^s \\
& + \Phi_{12} \bar{\Phi}_4 H_{16}^s H_{19}^s V_{17} V_{25} \\
& + \Phi_{12} \bar{\Phi}_4 H_{16}^s H_{19}^s V_{20} V_{29} \\
& + \Phi_{12} \Phi_{56} H_{30}^s H_{37}^s V_{19} V_{20} \\
& + \Phi_{12} \bar{\Phi}_{56} H_{18}^s H_{19}^s V_{11}^s V_{22}^s \\
& + \Phi_{13} \Phi_{23} H_{16}^s H_{38}^s V_2^s V_{11}^s \\
& + \Phi_{13} \Phi_{23} H_{18}^s H_{38}^s V_1^s V_{12}^s \\
& + \Phi_{23} \bar{\Phi}_4 H_{21}^s H_{30}^s V_2^s V_{21}^s \\
& + H_{16}^s H_{19}^s H_{38}^s V_{32}^s V_4 H_{35} \\
& + H_{21}^s H_{30}^s H_{31}^s H_{39}^s V_{12}^s V_{31}^s \\
& + \dots]
\end{aligned}$$

D Higgs Doublet, Quark, and Charged Lepton Mass Terms (through 9th order) for Flat Direction FDNA(5+8)

FDNA(5+8) fields with VEVs:

$$\Phi_{23}\Phi_4\bar{\Phi}_4\Phi'_4\bar{\Phi}'_4\bar{\Phi}'_{56}H_{15}^sH_{21}^sH_{30}^sH_{31}H_{37}^sH_{38}^sN_3^cH_{26}V_5V_{35}V_{37}$$

Additional VEVs allowed below FI scale: $\bar{\Phi}_{12} \sim 10^{-4}$

Higgs Doublet Mass Terms

$$\begin{aligned} h_2\bar{h}_1\bar{\Phi}_{12} &+ h_2\bar{h}_4H_{31}^s & +h_3\bar{h}_2\Phi_{23} \\ +h_4\bar{h}_3H_{38}^s &+ h_4\bar{h}_4\Phi_{23}H_{31}H_{38} & +h_1\bar{h}_3N_3^c\Phi'_4H_{15}H_{30}H_{31}H_{26}V_{37} \end{aligned} \quad (\text{D.1})$$

$\bar{h}Qu^c$ Terms

$$\begin{aligned} \bar{h}_1Q_1u_1^c &+ \bar{h}_4Q_3u_3^cH_{30}^s & +\bar{h}_3Q_1u_2^cH_{15}^sH_{30}^s \\ +\bar{h}_3Q_2u_2^c\Phi'_4H_{15}^sH_{30}^s &+ \bar{h}_4Q_1u_2^c\Phi_{23}H_{15}^sH_{30}^sH_{31} & +\bar{h}_3Q_1u_2^cN_3^c\bar{\Phi}'_{56}H_{30}^sH_{26}V_{37} \\ +\bar{h}_4Q_2u_1^c\Phi_{23}\bar{\Phi}'_4H_{15}^sH_{30}^sH_{31}^s &+ \bar{h}_3Q_2u_1^cN_3^c\bar{\Phi}'_{56}H_{30}^sH_{26}V_{37} & +\bar{h}_2Q_1u_2^cN_3^c\Phi_{23}\bar{\Phi}'_{56}H_{30}^sH_{26}V_{37} \end{aligned} \quad (\text{D.2})$$

hQd^c Terms

$$\begin{aligned} h_2Q_2d_2^c &+ h_3Q_3d_3^c & +h_4Q_1d_3^cH_{21}^s \\ +h_4Q_2d_2^c\Phi_{23}H_{38}^s &+ h_2Q_1d_2^cN_3^cH_{31}^sH_{26}V_{37} & +h_2Q_1d_2^c\Phi'_4H_{21}^sH_{30}^sV_5V_{35} \\ +h_2Q_3d_2^c\Phi'_4H_{30}^sH_{38}^sV_5V_{35} &+ h_3Q_1d_2^cN_3^c\Phi_{23}H_{31}^sH_{26}V_{37} & +h_2Q_2d_1^cN_3^c\Phi'_4\bar{\Phi}'_{56}H_{30}^sH_{26}V_{35} \\ +h_2Q_2d_3^c\Phi'_4\bar{\Phi}'_{56}H_{30}^sH_{38}^sV_5V_{35} &+ h_3Q_3d_2^c\Phi_{23}\bar{\Phi}'_4H_{30}^sH_{38}V_5V_{35} & +h_4Q_1d_2^cN_3^c\Phi_{23}H_{31}^sH_{38}H_{26}V_{37} \end{aligned} \quad (\text{D.3})$$

hLe^c Terms

$$\begin{aligned} h_2L_2e_2^c &+ h_3L_3e_3^c & +h_4L_2e_2^c\Phi_{23}H_{38}^s \\ +h_4L_3e_1^c\Phi_4H_{21}^s &+ h_4L_3e_1^c\Phi'_4\bar{\Phi}'_4H_{21}^s & +h_2L_2e_1^cN_3^c\bar{\Phi}'_{56}H_{30}^sH_{26}V_{35} \\ +h_2L_2e_3^c\Phi_4H_{30}^sH_{38}^sV_5V_{35} &+ h_2L_1e_2^cN_3^c\bar{\Phi}'_{56}H_{30}^sH_{26}V_{35} & +h_2L_2e_1^c\Phi_4\Phi_4H_{21}^sH_{30}^sV_5V_{35} \\ +h_2L_2e_1^c\Phi'_4\bar{\Phi}'_4H_{21}^sH_{30}^sV_5V_{35} &+ h_2L_3e_2^c\Phi_4\bar{\Phi}'_{56}H_{30}^sH_{38}^sV_5V_{35} & +h_3L_1e_2^cN_3^c\Phi_{23}\bar{\Phi}'_{56}H_{30}^sH_{26}V_{35} \\ +h_3L_2e_1^cN_3^c\Phi_{23}\bar{\Phi}'_4H_{31}^sH_{26}V_{37} &+ h_3L_2e_3^c\Phi_{23}\Phi_4H_{30}^sH_{38}^sV_5V_{35} \end{aligned} \quad (\text{D.4})$$

Note: no proton decay terms exist for this flat direction through at least 9th order.

Additional Higgs Doublet mass terms when $\langle\bar{\Phi}_{23}, \Phi'_{56}\rangle > 0$

$$h_2\bar{h}_3\bar{\Phi}_{23} \quad (\text{D.5})$$

Additional hQd^c mass terms when $\langle\bar{\Phi}_{23}, \Phi'_{56}\rangle > 0$

$$h_4Q_1d_2^c\Phi_{23}\Phi'_{56}H_{15}^sH_{31}^sH_{38}^s \quad +h_1Q_3d_3^c\Phi'_4\bar{\Phi}'_{56}H_{15}^sH_{30}^sH_{31}^s \quad (\text{D.6})$$

Additional hLe^c mass terms when $\langle \bar{\Phi}_{23}, \Phi'_{56} \rangle > 0$

$$h_1 L_3 e_3^c \Phi'_4 \Phi'_{56} H_{15}^s H_{15}^s H_{30}^s H_{31}^s + h_4 L_2 e_1^c \Phi_{23} \Phi'_4 \Phi'_{56} H_{15}^s H_{31}^s H_{38}^s \quad (D.7)$$

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